AchieveMath

Student Book

Volume 2

Name:

Catapult Learning™

Unit 3:

Unit Rates and Proportions

Catapult Learning™

Catapult Learning™

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Robot Racers

Use tape diagrams and equivalent ratios to find unit rates and answer questions.

After the first test run, students in the club adjusted their robots to improve their speeds. The results from their next set of speed tests are in the table shown.

Student	Robot	Distance Traveled (yards)	Time (minutes)
Camden	Turbo	28	4
Rashad	Super Cool	24	6
Tyreek	Booker	28.8	4.8

1. Write each robot's speed as a ratio of distance to time.

Turbo: _____ yards/_____ minutes

Super Cool: _____ yards/_____ minutes

Booker: _____ yards/_____ minutes

2. Use the tape diagram to find Turbo's unit rate.

a. How many yards did Turbo travel in 1 minute? _____

b. Turbo's unit rate is _____ yards/minute.

3. Use equivalent ratios to find Super Cool's unit rate.

Super Cool's unit rate is _____ yards/minute.

4. Use equivalent ratios to find Booker's unit rate.

Booker's unit rate is _____ yards/minute.

Bot Basketball

Review the example problem. Use equivalent ratios to find unit rates and answer questions. Show your work.

Example

Shelly's robot, Blaze, competed in the basketball challenge. Blaze threw a basketball through a hoop to score points. After **8** minutes, Blaze made **24** baskets. What is Blaze's unit rate of baskets per minute?

Step 1	Step 2	Step 3
Write a ratio of baskets to minutes. **24** baskets:**8** minutes $= \frac{\mathbf{24}}{\mathbf{8}}$	Set up the equation to find the unit rate. Divide both parts of the ratio by the value of the minutes to get the number of baskets in 1 minute. $\frac{\mathbf{24}}{\mathbf{8}} \div \frac{\mathbf{8}}{\mathbf{8}} = ?$	Use equivalent rates to find unit rate. $\frac{\mathbf{24}}{\mathbf{8}} \div \frac{\mathbf{8}}{\mathbf{8}} = \frac{3}{1}$ The unit rate is $\frac{3}{1}$ or 3 baskets per minute.

1. Super Cool made 72 baskets in 6 minutes. What is its unit rate per minute?

_____ baskets per minute

2. Terminator made 16 baskets in 4 minutes. What is its unit rate per minute?

_____ baskets per minute

3. Axel made 20 baskets in 5 minutes. What is its unit rate per minute?

_____ baskets per minute

4. Champ made 56 baskets in 8 minutes. What is its unit rate per minute?

_____ baskets per minute

5. Blockbuster made 28 baskets in 3.5 minutes. What is its unit rate per minute?

_____ baskets per minute

6. Scrappy's batteries ran out at 3.75 minutes. Scrappy had made 15 baskets in that time. What is its unit rate per minute?

_____ baskets per minute

Lesson 17 Exit Ticket

Use tape diagrams and equivalent rates to find unit rates and answer questions.

1. The concession stand at the robotics competition sold lunch combos. During the lunch break, the stand sold 5 combos for a total of $17.50.

a. Write a ratio to represent the rate of dollars for lunch combos sold.

_____

b. Use the tape diagram to find the unit rate per lunch combo. Show your work.

The unit rate for a lunch combo is _____.

2. To make the lemonade, Mercedes used 22.5 cups of water with 4.5 scoops of lemonade mix. Use equivalent ratios to find the equivalent unit rate per scoop. Show your work.

The unit rate is _____ cups of water per scoop of mix.

3. There are 40 hot dogs in 5 packages. Use equivalent ratios to find the unit rate of hot dogs per package. Show your work.

The unit rate is _____ hot dogs per package.

Extra Practice: Robot Ribbons

Use tape diagrams and equivalent ratios to find unit rates and answer questions.

Ms. Gupta purchased ribbons for the winners in the different events at a chess competition. It cost $6 for a package of 12 ribbons.

1. Use the tape diagram to find the unit rate for cost in dollars per ribbon.

The unit rate is _____ dollars per ribbon.

2. Find the unit rate using equivalent ratios. Show your work.

The unit rate is _____ dollars per ribbon.

3. Which process do you prefer to use to find the unit rate? Explain your choice.

_____

_____

Match each rate with the correct unit rate.

$\frac{10.5}{2.5}$	$\frac{9}{1}$
$\frac{36}{4}$	$\frac{8}{1}$
$\frac{16}{5}$	$\frac{4}{1}$
$\frac{18}{4.5}$	$\frac{3.2}{1}$
$\frac{56}{7}$	$\frac{4.2}{1}$

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Storm Speeds

Model each ratio with a **tape diagram**. Then find the unit rate.

1. Once it made landfall, Hurricane Khloe slowed to a storm and traveled $11\frac{1}{12}$ of a mile every $\frac{1}{8}$ of an hour.

a. Write Hurricane Khloe's rate as a complex fraction ratio. _____

b. How many sections is each tape of the tape diagram divided into? _____ Why?

_____

_____

c. What is the value of each part in the top tape? _____ Why? _____

_____

d. Add or multiply to find the total number of miles shown by the tape diagram.

Hurricane Khloe's unit rate is _____ miles per hour.

2. Once it made landfall, Hurricane Brandy slowed to a storm and traveled $15\frac{15}{16}$ of a mile every $\frac{1}{12}$ of an hour.

a. Write Hurricane Brandy's rate as a complex fraction ratio. _____

b. How many sections is each tape of the tape diagram divided into? _____

c. What is the value of each part in the top tape? _____

d. Add or multiply to find the total number of miles shown by the tape diagram.

Hurricane Brandy's unit rate is _____ miles per hour.

Pressure Change

Review the example problem. Then model each ratio with a **tape diagram** and find the unit rate.

Example

When a storm approaches, the barometric pressure drops. As the storm closed in, Santana saw that the pressure went down **½** of a millibar every **⅓** of an hour. What is the unit rate for the pressure drop?

Step 1

Write the rate as a ratio using a complex fraction.

$$\frac{\frac{1}{2}}{\frac{1}{3}}$$

Step 2

Identify the fraction that needs to be changed to 1. Use the denominator to divide the tape diagram into equal parts.

⅓ hour needs to change to 1 hour, so the tape diagram should be divided into 3 parts.

Millibars

Hours

Step 3

Write the rate in the first column of the tape diagram.

The rate is **½** millibars every **⅓** hour.

Millibars

$\frac{1}{2}$		
$\frac{1}{3}$		

Hours

Step 4

Fill in the other parts of the tape diagram and add to find the unit rate.

Millibars

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Hours

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$$

The unit rate is $1\frac{1}{2}$ millibars per hour.

1. Find the unit rate for a decrease of ⅙ of a millibar every ¼ of an hour.

The unit rate is _____ millibars per hour.

2. Find the unit rate for a decrease of ⅛ of a millibar every ½ hour.

The unit rate is _____ millibars per hour.

3. Find the unit rate for a decrease of ¾ of a millibar every ⅓ of an hour.

The unit rate is _____ millibars per hour.

4. Find the unit rate for a decrease of ½ of a millibar every ⅙ of an hour.

The unit rate is _____ millibars per hour.

Lesson 18 Exit Ticket

Model each ratio with a tape diagram. Then find the unit rate.

1. A recipe calls for $\frac{1}{8}$ of a teaspoon of red food coloring for $\frac{1}{3}$ of a batch of cupcakes.

a. Write the rate of teaspoons per batches as a ratio using a complex fraction: _____

b. Use the tape diagram to model one batch of cupcakes.

c. The unit rate is _____ of a teaspoon of red food coloring per batch of cupcakes.

2. The frosting recipe called for $\frac{3}{4}$ of a teaspoon of vanilla for $\frac{1}{2}$ cup of sugar.

a. Write the rate of teaspoons per cups of sugar as a ratio using a complex fraction: _____

b. Use the tape diagram to model 1 cup of sugar.

c. The unit rate is _____ teaspoons of vanilla per cup of sugar.

Extra Practice: Mask Making

Use a tape diagram to find the unit rate.

1. Uma sews face masks to donate to her community so everyone can stay healthy during flu season. $\frac{3}{8}$ of a yard of elastic is needed to make $\frac{1}{3}$ of a mask. How much elastic does she need to make 1 mask?

Uma needs _____ yards of elastic per mask.

2. Uma uses $\frac{1}{6}$ of a yard of fabric to make $\frac{2}{3}$ of a mask. How many masks can she make per yard of fabric?

Uma can make _____ masks per yard of fabric.

3. Uma gives out $\frac{1}{5}$ of her masks in $\frac{1}{4}$ of an hour. How many masks will she give out in an hour?

Uma will give out _____ of her masks in 1 hour.

Tape Diagrams

Tape Diagrams

Fun Walk and Run

Max is also registered for the fun walk and run. Max has been doing training and keeping the rates in a notebook. A section of his notebook is shown. Use fraction division to find unit rates and answer questions.

Day	Distance Walked (miles)	Time (hours)
Monday	$\frac{5}{8}$	$\frac{4}{7}$
Tuesday	$\frac{2}{3}$	$\frac{3}{5}$
Wednesday	$2\frac{3}{4}$	$1\frac{1}{6}$
Thursday	$1\frac{1}{4}$	$\frac{4}{5}$

1. Write a ratio representing Max's Monday walk. _____ miles / _____ hours

2. How can you write this complex fraction as fraction division? _____

3. Explain how to solve the division equation in question 2. _____

_____

4. What was Max's unit rate on Monday? Show your work.

Max's unit rate on Monday was _____ miles per hour.

5. What was Max's unit rate on Thursday? Show your work.

Max's unit rate on Thursday was _____ miles per hour.

6. What were Max's unit rates on Tuesday and Wednesday? Show your work.

Tuesday: _____ miles per hour

Wednesday: _____ miles per hour

Wading Pool

Review the example problem. Then use fraction division to find unit rates. Show your work.

Example

After breaking a sweat, Max uses a garden hose to fill a wading pool he wants to cool off in. The water level in the pool rises $\frac{3}{8}$ of an inch in $\frac{1}{3}$ of a minute. What is the unit rate of the water level increase?

Step 1

Convert mixed numbers if you need to. Then write a ratio representing the water level rising $\frac{3}{8}$ of an inch every $\frac{1}{3}$ of a minute.

$$\frac{\frac{3}{8}}{\frac{1}{3}}$$

Rewrite the complex fraction ratio as a fraction division expression.

$$\frac{3}{8} \div \frac{1}{3}$$

Step 3

Find the reciprocal of the divisor.

Divisor = $\frac{1}{3}$

Reciprocal of $\frac{1}{3} = \frac{3}{1}$

Step 4

Multiply the dividend by the reciprocal to solve.

$$\frac{3}{8} \div \frac{1}{3} = \frac{3}{8} \times \frac{3}{1} = \frac{9}{8} = 1\frac{1}{8}$$

The water level rises at a unit rate of 1⅛ inches per minute.

1. The water increases ⅛ of an inch every ¾ of a minute.

Unit rate: _____ inches per minute

2. The water increases 7/10 of an inch every 1 ⅔ minutes.

Unit rate: _____ inches per minute

3. The water increases 2 ¾ inches every 3 ⅘ minutes.

Unit rate: _____ inches per minute

4. The water increases ¾ of an inch every ⅞ of a minute.

Unit rate: _____ inches per minute

Lesson 19 Exit Ticket

Find the unit rate for the given ratios. Show your work.

1. $\dfrac{4\frac{1}{2} \text{ miles}}{1\frac{9}{10} \text{ hours}}$

Unit rate: _____

2. $\dfrac{\frac{2}{5} \text{ miles}}{\frac{4}{15} \text{ hours}}$

Unit rate: _____

3. $\dfrac{\frac{7}{12} \text{ miles}}{\frac{3}{4} \text{ hours}}$

Unit rate: _____

4. $\dfrac{5\frac{4}{5} \text{ miles}}{2\frac{1}{3} \text{ hours}}$

Unit rate: _____

Extra Practice: Creeping Around

Part 1: Determine whether the following statements are *true* or *false*.

1. The reciprocal of a unit fraction is always a whole number. _____

2. The first step in finding the unit rate for 2 ½ inches per ⅗ of a minute is to convert the mixed number to an improper fraction. _____

3. A unit rate can never be greater than 1. _____

4. A unit rate is a rate in which one of the quantities is set to 1. _____

Part 2: Use fraction division to find unit rates with complex fractions and answer questions.

5. Addie planted Virginia creeper in her side yard. Then she put down fertilizer to increase the growth rate. Her plants grew 8 ¾ inches in ⅗ of a year.

a. Write a ratio representing the growth in inches to years. _____

b. How can you write this complex fraction as a fraction division equation? _____

c. What is the reciprocal of ⅗? _____

d. What is the unit rate for the Virginia creeper's growth? Show your work.

Addie's Virginia creeper has a growth rate of _____ inches per year.

Part 3: Find the unit rate for the given ratios. Show your work.

6. $\dfrac{\frac{5}{6} \text{ inches}}{\frac{3}{4} \text{ year}}$

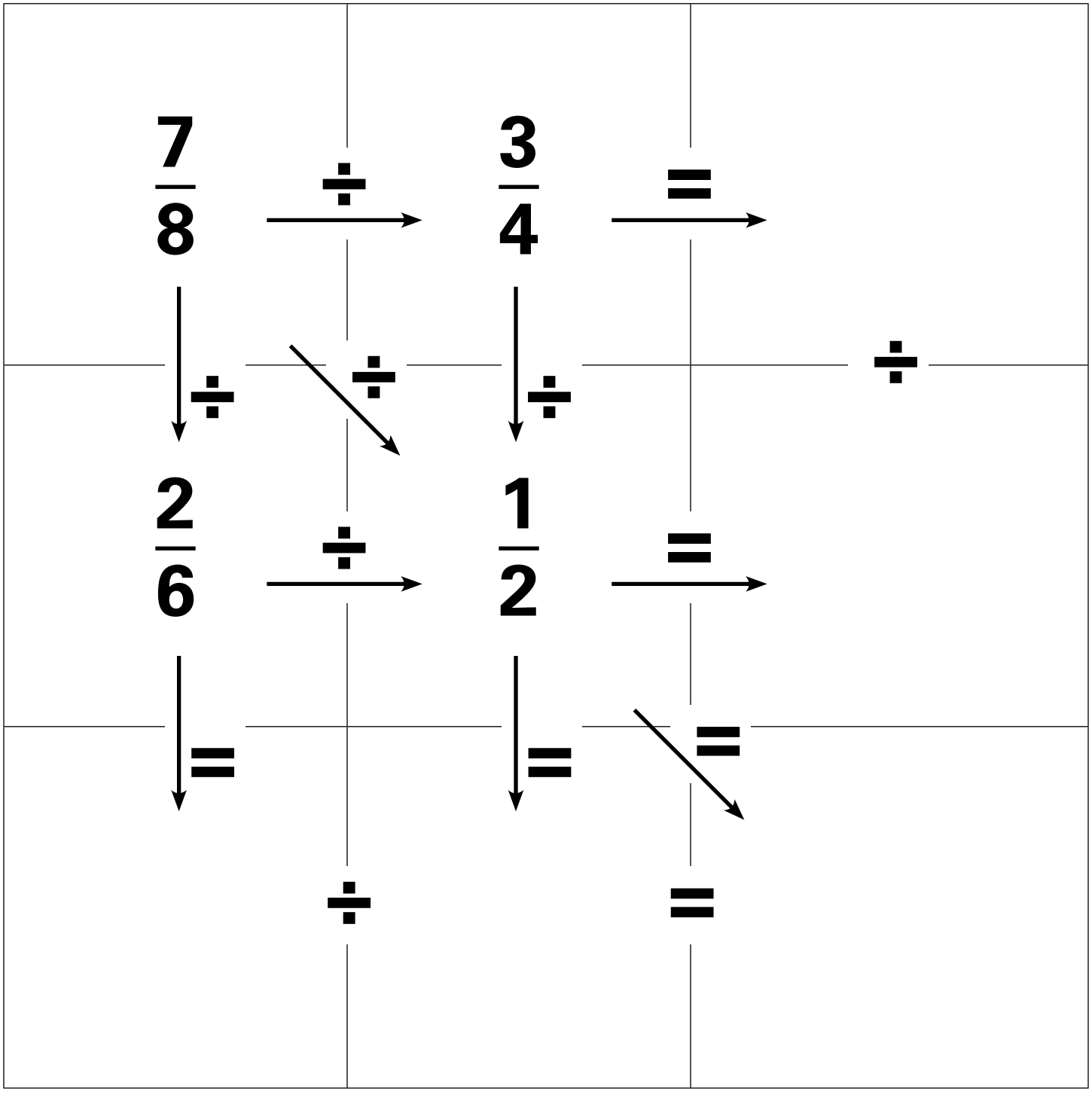
Unit rate: _____

7. $\dfrac{\frac{1}{7} \text{ inches}}{\frac{1}{9} \text{ year}}$

Unit rate: _____

Fraction Division 9-Square

Write the unit rate for each row and column of the grid.



Snail's Pace

Out in the forest, Kendra found three different kinds of snails. She gave each snail a name and measured its speed. She recorded the information in the table as shown. Use a tape diagram to find equivalent ratios and determine if different rates are proportional. Then, answer the questions.

Snail	Distance (meters)	Time (hours)
Quicksilver	2	8
Zippy	1	4
Lightning	8	12

1. Which snail's rate uses the smallest numbers? _____

2. Draw a tape diagram to model Zippy's speed.

a. Extend your tape diagram to show if Zippy's speed is proportional to Quicksilver's speed. Are Zippy and Quicksilver's speeds proportional? _____ Explain how you know using each snail's rate._____

_____

_____

b. Extend your tape diagram to show if Zippy's speed is proportional to Lightning's speed. Are Zippy and Lightning's speeds proportional? _____ Explain how you know using each snail's rate._____

_____

_____

River Flow

Review the example problem. Then draw **tape diagrams** to determine whether the given rates are proportional.

Example

Ibrahim measured the flow rates of two creeks near his house. Goose Creek flowed at a speed of **6** feet in **18** seconds. Oak Creek flowed at a speed of **2** feet in **6** seconds. Are these rates proportional?

Step 1

Determine which rate has the lesser quantities and represent it with a tape diagram.

Oak Creek's rate has the lesser quantities: **2** feet in **6** seconds.

Feet

2
6

Seconds

Step 2

Multiply each tape in the diagram to find an equivalent rate with a same measurement as the other rate.

Multiply **2** feet 3 times to see how long it takes Oak Creek to go **6** feet, the distance Goose Creek flowed.

Feet

2	**2**	**2**
6	**6**	**6**

Seconds

Step 3

Write the equivalent rate that the extended tape diagram represents.

The tape diagram shows that Oak Creek will flow **6** feet in **18** seconds.

Step 4

Compare the equivalent rate to the given rate with the greater quantities.

Oak Creek will flow **6** feet in **18** seconds. Goose Creek flows **6** feet in **18** seconds. The rates are equivalent, so they are proportional.

1. Are 2:5 and 6:20 proportional? _____

2. Are 3:4 and 15:20 proportional? _____

3. Are 5:8 and 40:64 proportional? _____

4. Are 2:11 and 12:55 proportional? _____

5. Are 4:5 and 20:30 proportional? _____

6. Are 8:9 and 48:54 proportional? _____

7. Are 3:10 and 18:60 proportional? _____

8. Are 1:2 and 10:24 proportional? _____

9. Are 1:3 and 5:15 proportional? _____

10. Are 3:5 and 21:45 proportional? _____

Lesson 20 Exit Ticket

Read the problems. Then use the information in the table to model the rates with tape diagrams to determine whether they are proportional.

Animal	Distance (miles)	Time (hour)
Tommy Tortoise	2	4
Marti Manatee	8	10
Sammy Starfish	1	9
Susie Sloth	3	27

1. Are Tommy Tortoise and Marti Manatee's speeds proportional? _____

2. Are Sammy Starfish and Susie Sloth's speeds proportional? _____

Extra Practice: Obstacle Course

Part 1: After they completed the military crawl section of the obstacle course, Travis and Lydia had to climb up a net wall. Travis climbed at a rate of 4 feet in 6 minutes. Lydia climbed at a rate of 9 feet in 12 minutes. Use tape diagram models to find equivalent ratios and determine whether the different rates are proportional. Then, answer the questions.

1. Draw tape diagrams to model Travis's rate and Lydia's rate.
Travis:

Lydia:

2. How can you use the tape diagrams to determine if Travis's and Lydia's rates are proportional?

_____

_____

_____

_____

3. Are the climbing rates for Travis and Lydia proportional? _____

Part 2: Draw tape diagrams to find equivalent ratios. Circle the rates that are proportional. Draw a line through the rates that are not proportional.

$\frac{6}{8}$ and $\frac{3}{4}$

$\frac{3}{5}$ and $\frac{9}{15}$

$\frac{2}{5}$ and $\frac{6}{10}$

$\frac{3}{6}$ and $\frac{5}{12}$

$\frac{1}{4}$ and $\frac{5}{20}$

$\frac{5}{8}$ and $\frac{2}{4}$

$\frac{2}{9}$ and $\frac{4}{18}$

$\frac{3}{8}$ and $\frac{5}{16}$

$\frac{5}{7}$ and $\frac{9}{14}$

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Woo-hoo, Goo!

Jed works with Keisha and is interested in developing a new recipe for goo. He asks some of the students for help. Each student used the amount of food coloring and glue shown in the table. Use unit rates and ratio tables to determine whether different rates are proportional. Then, answer the questions.

Student	Food Coloring (drops) (x)	Glue (cups) (y)
Flynn	4	8
Amaris	3	9
Reece	6	18

1. What is the independent variable in this scenario? How do you know?

_____

_____

_____

2. Write the ratios of cups of glue to food coloring drops for the goo each student makes.

Flynn: _____ Amaris: _____ Reece: _____

3. Jed decides the best recipe is 6 cups of glue for every 2 drops of food coloring. What is the unit rate for Jed's recipe?

The unit rate is _____ cups per _____ drop.

4. Complete the ratio table for Jed's recipe.

5. Which of the students use amounts that are proportional to Jed's recipe?

_____

Jed's Recipe

Food Coloring (drops) (x)	Glue (cups) (y)
1	
2	
3	
4	
5	
6	

Sidewalk Chalk

Review the example problem. Then find the unit rates and use **ratio tables** to determine whether the given rates are proportional.

Example

Keisha's students make colored sidewalk chalk. The chalk recipe calls for **8** cups of plaster for every **2** cups of paint used. Miles makes his chalk with **16** cups of plaster and **4** cups of paint. Is his ratio of plaster to paint proportional to the recipe?

Step 1

Identify the independent variable. Fill in a ratio table showing the independent variable in the *x* column.

Since the amount of plaster depends on how much paint is being used, the independent variable is the cups of paint

Paint (cups) (*x*)	Plaster (cups) (*y*)
1	
2	
3	
4	

Step 2

Write the ratios for the given amounts in cups of paint to cups of plaster. Write the independent variable last in the ratio.

Recipe ratio: **8:2**
Miles's recipe: **16:4**

Step 3

Find the unit rate for one of the ratios. Divide each quantity in the ratio by a factor that will make the independent variable equal 1.

$$\frac{\mathbf{8}}{\mathbf{2}} \div \frac{2}{2} = \frac{4}{1}$$

Step 4

Use the unit rate and multiplication to complete the ratio table.

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12} \qquad \frac{1}{4} \times \frac{4}{4} = \frac{4}{16}$$

Miles's ratio, $^{16}\!/_{4}$, appears in the table, so the ratio is equivalent and proportional to the recipe ratio.

Paint (cups) (*x*)	Plaster (cups) (*y*)
1	4
2	**8**
3	12
4	**16**

1. 25 pieces of chalk per 5 students
8 pieces of chalk per 2 students

What is the independent variable?

_____

Are the ratios proportional? _____

2. 2 students per 60 feet of sidewalk
5 students per 150 feet of sidewalk

What is the independent variable?

_____

Are the ratios proportional? _____

Lesson 21 Exit Ticket

In the after-school program, 25 students are allowed to sign up per every 5 staff members.

This afternoon, 40 students sign up. There are 7 staff members. Use unit rates and ratio tables to determine whether the rates are proportional.

1. What is the ratio of students allowed to sign up per staff members?_____

2. What is the unit rate for students allowed to sign up per staff member? _____

3. What is the independent variable in this scenario?

_____

4. Complete the ratio table to show the allowed number of students to staff members.

5. Is the rate 40 students per 7 staff members represented on the ratio table? _____What does that mean?

_____

Staff (x)	Students (y)
1	
2	
3	
4	
5	
6	
7	
8	

Extra Practice: Photo Posters

Part 1: Tania's print shop just got a new machine, so she can now make posters in ratios proportional to 12 feet long per 3 feet wide. Use unit rates and ratio tables to determine whether the rates are proportional. Then, answer the questions.

1. What is the independent variable in the ratio? How do you know?

_____

_____

2. What is the ratio for the posters Tania can now make in length to width? _____

3. What is the unit rate for the ratio? Show your work.

The new unit rate for posters is 4 feet long for every _____ foot wide.

4. Complete the ratio table for poster dimensions.

5. A customer wants a poster 5 feet wide and 25 feet long. Is this ratio proportional to the ratio the machine makes?

_____

Width (feet) (*x*)	Length (feet) (*y*)
1	
2	
3	
4	
5	
6	

Part 2: Complete the ratio table.

Water (teaspoons) (*x*)	Sugar (tablespoons) (*y*)
2	16
	24
4	
5	
6	
	56
	64
9	
	80

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Cooking Class

In cooking class, Riley was given a table with ratios of cups of milk to tablespoons of cocoa powder to make hot chocolate. He needs to check if the recipes are proportional. Answer the questions about the ratios.

Recipe	Milk (cups)	Cocoa Powder (tablespoons)
A	$1\frac{1}{2}$	$\frac{3}{4}$
B	3	$1\frac{1}{2}$
C	$4\frac{1}{2}$	$2\frac{1}{4}$
D	6	$3\frac{1}{8}$

1. Write the ratios for each recipe as a complex fraction.

Recipe A: _____ Recipe B: _____ Recipe C: _____ Recipe D: _____

2. Using cross multiplication, check whether each pair of recipes is proportional. Show your work.

a. Are Recipes A and B proportional? _____

b. Are Recipes B and C proportional? _____

c. Are Recipes C and D proportional? _____

More Cooking

Review the example problem. Then use cross multiplication to determine whether the ratios are proportional.

Example

Riley found a recipe that uses **6** tablespoons of butter to make **48** cookies. He only has **3** tablespoons of butter. He thinks the proportional number of cookies he can make is **24**. Is he correct?

Step 1

Write the rates for the given quantities.

Recipe: $\frac{\mathbf{6}}{\mathbf{48}}$ Riley: $\frac{\mathbf{3}}{\mathbf{24}}$

Step 2

Write an equation with the two ratios.

$\frac{\mathbf{6}}{\mathbf{48}} = \frac{\mathbf{3}}{\mathbf{24}}$

Step 3

Circle the numerator of the first ratio and the denominator of the second ratio.

$\frac{\mathbf{6}}{\mathbf{48}} = \frac{\mathbf{3}}{\mathbf{24}}$

Circle the numerator of the second ratio and the denominator of the first ratio.

$\frac{\mathbf{6}}{\mathbf{48}} = \frac{\mathbf{3}}{\mathbf{24}}$

Step 4

Cross multiply the ratios.

Multiply each set of circled numbers. If the products are equal, the rates are proportional.

$\mathbf{6} \times \mathbf{24} = \mathbf{3} \times \mathbf{48}$
144 = 144

The products are equal, so the rates are proportional.

1. Are 6:48 and 96:768 proportional? _____

2. Are 1:3 and 5:12 proportional? _____

3. Are $\frac{4}{9} : \frac{2}{3}$ and $\frac{1}{9} : \frac{1}{6}$ proportional? _____

4. Are $\frac{2}{3} : \frac{1}{6}$ and $\frac{5}{2} : \frac{5}{8}$ proportional? _____

Lesson 22 Exit Ticket

Use cross multiplication to determine whether the ratios are proportional.

1. Are $\frac{3}{8}:\frac{2}{16}$ and $\frac{3}{2}:\frac{2}{4}$ proportional? _____

2. Are 13:15 and 55:59 proportional? _____

3. Are $\frac{3}{4}:\frac{1}{2}$ and $1:\frac{2}{3}$ proportional? _____

4. Are $\frac{4}{7}$ and $\frac{6}{21}$ proportional? _____

Extra Practice: Donut Day

Part 1: Use cross multiplication to determine whether the ratios are proportional. Show your work.

1. Joaquin can make 45 donuts in ¾ of an hour. Ashlee can make 135 donuts in 2 ¼ hours. Are Joaquin's and Ashlee's donut-making speeds proportional? _____

2. Kurt can make 21 donuts in ⅚ hours. Ella can make 126 donuts in 2 ⅙ hours. Are Kurt and Ella's donut-making speeds proportional? _____

3. Sarah can make 29 donuts in ⅘ hours. Gabriel can make 23 donuts in ⅔ hours. Are Sarah and Gabriel's donut-making speeds proportional? _____

Part 2: Determine whether the following statements are true or false.

4. To cross multiply, multiply the ratios' numerators together and their denominators together. _____

5. Equivalent ratios are proportional. _____

6. Cross multiplication is a helpful way to determine if ratios with complex fractions are proportional. _____

7. Ratios are proportional if the cross products are the same. _____

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Assessment

Unit 3 Assessment

1. Skylar participated in the school bike-a-thon. He bicycled 48 miles in 4 hours. Use the tape diagram to find Skylar's unit rate in miles per hour.

Skylar's unit rate is _____ miles per hour.

2. Esteban bought 15 bananas for $11.25. Find the unit rate using equivalent ratios. Show your work.

The unit rate is $_____ per banana.

3. Yesterday, it snowed $\frac{5}{8}$ of an inch in $\frac{1}{4}$ hours.

a. Write the inches of snow to hours rate as a complex fraction ratio.

b. Use the tape diagram to show yesterday's snow rate and to find the unit rate in inches per hour.

Yesterday's snow unit rate was _____ inches per hour.

4. Find the unit rate for the given rate. Show your work.

$$\frac{\frac{8}{9}}{\frac{2}{3}}$$

The unit rate is _____.

5. Trinh was filling a large pot with water from the faucet to make some vegetable soup. The level of water in the pot increases 1 ⅓ inches in ½ of a minute.

a. Write the water level increase rate as a ratio using a complex fraction. _____

b. Using fraction division, find the rate the water increases per minute. Show your work.

The water level increases at a unit rate of _____ inches per minute.

6. Shelly was grocery shopping yesterday. She compared two different brands of dog food for her puppy, Duke. Puppy Pal cost $15 for 6 pounds and Doggie-Bites cost $45 for 18 pounds.

a. Draw tape diagrams to model the rate for Puppy Pal dog food, up to 18 pounds.

b. Are the price per pound rates for the two brands of dog food proportional? _____

7. Find the unit rate. Then complete the ratio table.

The unit rate for teaspoons of yeast to cups of flour is _____.

Flour (cups) (*x*)	Yeast (teaspoons) (*y*)
2	6
3	
	12
	15
6	

8. Sung-ho reads 24 books per 6 weeks.

a. What is the independent variable in the ratio? _____

b. Miriam reads 10 books per 2 weeks. Is her reading rate proportional to Sung-ho's? Show your work on the ratio table and circle your answer.

_____ (*x*)	_____ (*y*)

Is Miriam's rate proportional to Sung-ho's rate? yes / no

9. Lila has a muffin recipe that uses $\frac{2}{3}$ of a teaspoon of cinnamon for $\frac{1}{2}$ of a teaspoon of vanilla extract. Lila has decided to make a really large batch of muffins and uses $\frac{10}{3}$ teaspoons of cinnamon and $\frac{5}{2}$ teaspoons of vanilla extract. Use cross multiplication to determine whether Lila's measurements are proportional to the original recipe. Show your work and circle your answer.

Is Lila's recipe proportional to the original recipe? yes / no

10. Parker read 86 pages in 4 days. Daria read 62 pages in 3 days. Are Parker's and Daria's reading rates proportional? Use cross multiplication and show your work. Circle your answer.

Is Daria's rate proportional to Parker's rate? yes / no

Cumulative Review

1. At Food Town, 4 cans of soup cost $5.

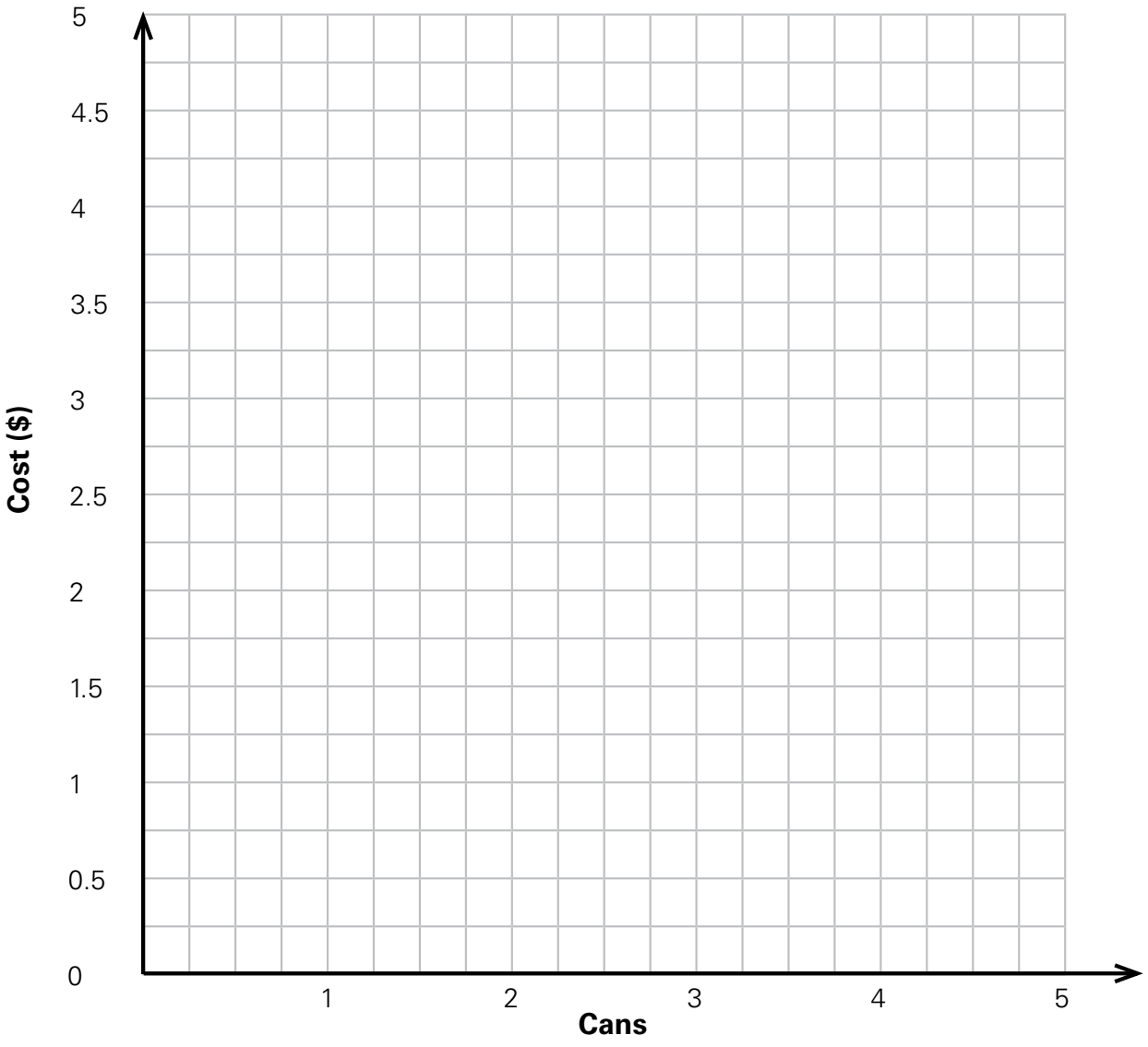
a. Write a ratio for the cost to the number of cans. _____

b. Complete the ratio table.

Cans	Cost ($)
1	
2	
3	
4	5.00

c. Write an equation to represent this situation. _____

d. Graph the equation on a coordinate plane.



2. Divide. Show your work.

$\frac{7}{8} \div \frac{4}{6} =$ _____

3. Eileen makes beaded bracelets and sells them online. She can make 7/12 of a bracelet with 1/4 of a bag of beads. What is the unit rate of bracelets per bag of beads? Show your work.

Eileen can make _____ bracelets per bag of beads.

4. Add.

$-36 + -15 =$ _____

5. Group like terms and use the distributive property to write an equivalent expression.

$21x + 32 + 15x + 13 =$ _____

6. Serena works in the box office at a movie theater. She notices that for every 10 adult tickets sold, 3 children's tickets are sold. Write each ratio 3 different ways.

Adult tickets to children's tickets: _____ _____ _____

Children's tickets to total tickets: _____ _____ _____

Total tickets to adult tickets: _____ _____ _____

7. Each hamburger costs $5. Angie spent $60 on hamburgers.

a. Use the variable x to write a multiplication equation to represent the number of hamburgers Angie bought.

_____

b. Angie bought _____ hamburgers.

8. Match each fraction with the correct decimal.

$\frac{16}{50}$	0.32
$\frac{9}{11}$	$0.1\overline{3}$
$\frac{2}{15}$	0.75
$\frac{1}{3}$	$0.\overline{81}$
$\frac{24}{32}$	$0.\overline{3}$

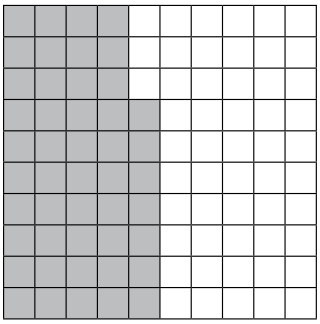
9. Circle the ratios that are proportional.

$^{18}/_{12}$ and $^{9}/_{2}$ $^{1}/_{2}$ and $^{50}/_{100}$

10. Write 4^3 in standard form. Show your work.

4^3 = _____

11. Write the percent for the grid.



_____%

12. Multiply.

$-12 \times -8 =$ _____

13. Tomas and Gigi are working with the inequality $p < 27$. Gigi says that 27 is part of the solution set. Tomas says it isn't. Who is correct? Explain. _____

_____

_____

14. Rayana earned $108 dog walking for 9 hours. What is Rayana's unit rate per hour?

$_____ per hour

15. Subtract.

$10 - 18 =$ _____

Unit 4:

Represent Proportional Relationships

Catapult Learning™

Snack Socks

Part 1: Find the constant of proportionality and complete a **ratio table** for each rate.

1. The curry-spiced popcorn recipe uses 4 teaspoons of curry powder per 5 cups of popped popcorn. What is the constant of proportionality? Show your work.

The constant of proportionality is _____.

2. The lemon-zest popcorn recipe uses 1 teaspoon of lemon zest per 4 cups of popped popcorn. What is the constant of proportionality? Show your work.

The constant of proportionality is _____.

Part 2: Identify the constant of proportionality in each ratio table.

3.

Cranberries (cups) (x)	Brown Sugar (teaspoons) (y)
0	0
1	$\frac{4}{3}$
2	$\frac{8}{3}$
3	4

The constant of proportionality is _____.

4.

Banana Chips (cups) (x)	Walnuts (teaspoons) (y)
0	0
1	3
2	6
3	9

The constant of proportionality is _____.

Purple Cake

Review the example problem. Then use the constant of proportionality to create **ratio tables**.

Example

Zoe wants a cake with purple frosting. She needs to use **3** teaspoons (tsp) of purple food coloring per **4** ounces (oz) of frosting. How many teaspoons of coloring will she need for smaller amounts of frosting?

Step 1

Set up a ratio table. The ounces of frosting is the independent variable. Fill in the ratio table with the values you already know.

The given ratio is **3** teaspoons of purple food coloring per **4** ounces of frosting.

Frosting (oz) (*x*)	Coloring (tsp) (*y*)
0	0
1	
2	
3	
4	**3**

Step 2

Use the given ratio to find the constant of proportionality, or the value of y when $x = 1$.

$$\frac{\mathbf{3}}{\mathbf{4}} = \frac{y}{1}$$

$$\frac{\mathbf{3}}{\mathbf{4}} \div \frac{4}{4} = \frac{3/4}{1} = \frac{3}{4}$$

The constant of proportionality is $\frac{3}{4}$.

Step 3

Multiply the ounces of frosting (x) by the constant of proportionality to find each value of y. Use the values of y to complete the ratio table.

$$y = 1 \times \frac{3}{4} = \frac{3}{4}$$

$$y = 2 \times \frac{3}{4} = \frac{6}{4}$$

$$y = 3 \times \frac{3}{4} = \frac{9}{4}$$

Frosting (oz) (*x*)	Coloring (tsp) (*y*)
0	0
1	3/4
2	6/4
3	9/4
4	**3**

1. **Rate:** 4 teaspoons of coloring per 10 ounces of frosting

   Constant of proportionality: _____

2. **Rate:** ½ teaspoon of coloring per 2 ounces of frosting

   Constant of proportionality: _____

3. **Rate:** 12 teaspoons of coloring to 5 ounces of frosting

   Constant of proportionality: _____

Lesson 24 Exit Ticket

Part 1: Find the constant of proportionality and complete the ratio table.

1. Priyanka is in charge of mixing the fruit punch at the school carnival. The container says to mix 8 teaspoons of powder with 2 cups of water.

What is the constant of proportionality? Show your work.

The constant of proportionality is _____.

Water (cups) (x)	Powder (teaspoons) (y)
0	
1	
2	

Part 2: Identify the constant of proportionality in each ratio table.

2.

Water (cups) (x)	Powder (teaspoons) (y)
0	0
1	5
2	10
3	15
4	20

The constant of proportionality is _____.

3.

Water (cups) (x)	Powder (teaspoons) (y)
0	0
1	$\frac{6}{7}$
3	$2\frac{4}{7}$
5	$4\frac{2}{7}$
7	6

The constant of proportionality is _____.

Extra Practice: Exercise Plan

Use the given ratios to fill in a ratio table. Then find the constant of proportionality.

1. Kevin's exercise plan has him complete 9 sit-ups for every 3 push-ups.

Push-ups (x)	Sit-ups (y)
0	0

The constant of proportionality is _____.

2. Peyton's exercise plan has her complete 2 minutes of jogging for every 5 squats.

Squats (x)	Jogging (minutes) (y)
0	0

The constant of proportionality is _____.

3. Brooklyn's exercise plan has her complete 1 lap around the track for every 3 jumping jacks.

Jumping Jacks (x)	Laps (y)
0	0

The constant of proportionality is _____.

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Craft Projects

Use a **coordinate plane** to graph each proportional relationship and answer the questions.

1. The students in art class need 4 yards of string to make 2 necklaces.

a. What is the constant of proportionality? _____

b. Complete the ratio table based on your graph.

(*x*)	(*y*)
0	0

2. The students in art class need 9 yards of fabric to make 3 pillows.

a. What is the constant of proportionality? _____

b. Complete the ratio table based on your graph.

(*x*)	(*y*)
0	0

3. The students in art class need 8 balls of yarn to make 2 blankets.

a. What is the constant of proportionality? _____

b. Complete the ratio table based on your graph.

(*x*)	(*y*)
0	0

The Art Room

Review the example problem. Then use a **coordinate plane** to find the constant of proportionality.

Example

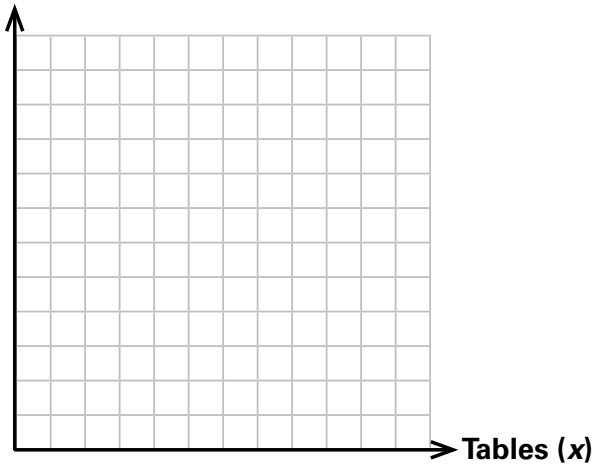
The art room has large tables for crafting. **12** students can sit for every **2** tables in the room. The art teacher wants to show the relationship between students and tables in a graph.

Step 1

Identify the independent (*x*) and dependent (*y*) variables and label the axes of the graph.

Since the number of students who can sit depends on the number of tables, the number of tables is the independent variable, and the number of students is the dependent variable.

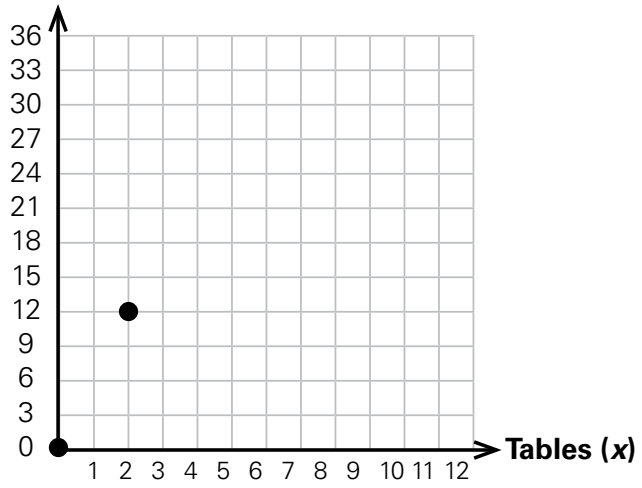
Students (*y*)



Step 2

Plot the origin (0, 0) and the given ratio (**2, 12**) on the graph.

Students (*y*)



Step 3

Use a ruler to draw a line through the points. Find the constant of proportionality, which is the *y* value when $x = 1$.

When *x* equals 1, *y* equals 6, so the constant of proportionality is 6.

1. What is the constant of proportionality for the relationship between 15 sheets of paper per 3 art mats?

_____

2. What is the constant of proportionality for the relationship between 12 sheets of paper per 3 art mats?

_____

3. What is the constant of proportionality for the relationship between 18 sheets of paper per 3 art mats?

_____

4. What is the constant of proportionality for the relationship between 10 sheets of paper per 5 art mats?

_____

Lesson 25 Exit Ticket

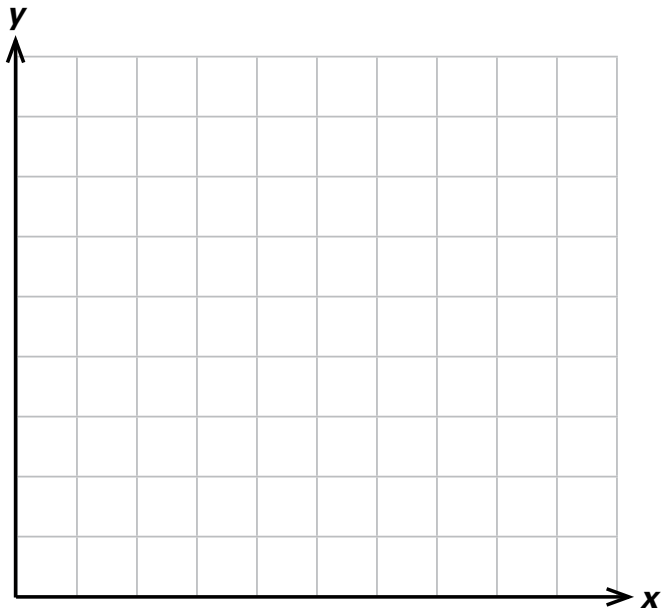
Use a coordinate plane to graph the proportional relationship and answer the questions.

To make a sand jar, fill a small glass jar with different layers of colored sand. The result looks like a painted desert in a jar! The directions say to make 4 jars, you need 24 packages of colored sand.

1. Create a graph to represent the proportional relationship.

2. What is the constant of proportionality? _____

3. Complete a ratio table based on your graph.



(*x*)	(*y*)

Extra Practice: Proportional Relationships

Part 1: Complete each statement by writing *always*, *sometimes*, or *never* in order to make the statement true.

1. The graph of a proportional relationship _____ goes through the origin.

2. In a ratio table, the constant of proportionality is _____ equal to the *y*-value when $x = 1$.

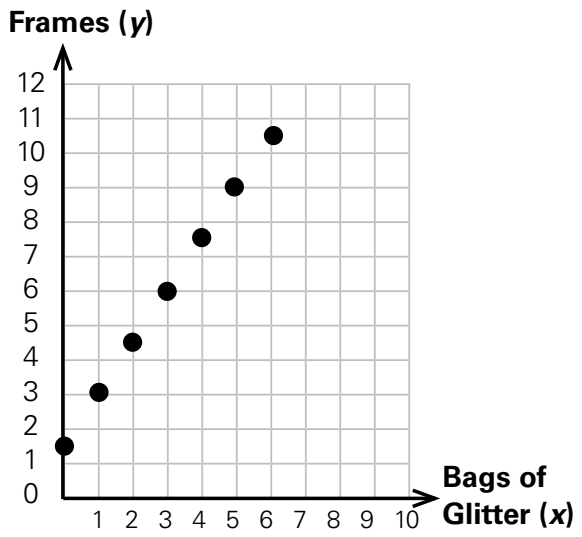
3. The line representing a proportional relationship is _____ curved.

4. The constant of proportionality of a proportional relationship _____ has the same value on a coordinate graph as it does on a related ratio table.

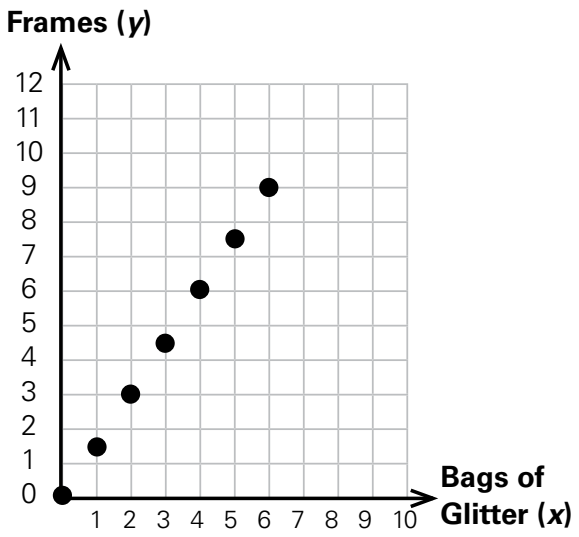
5. The constant of proportionality is _____ 1.

Part 2: Look at the graphs and answer the questions.

Graph 1



Graph 2



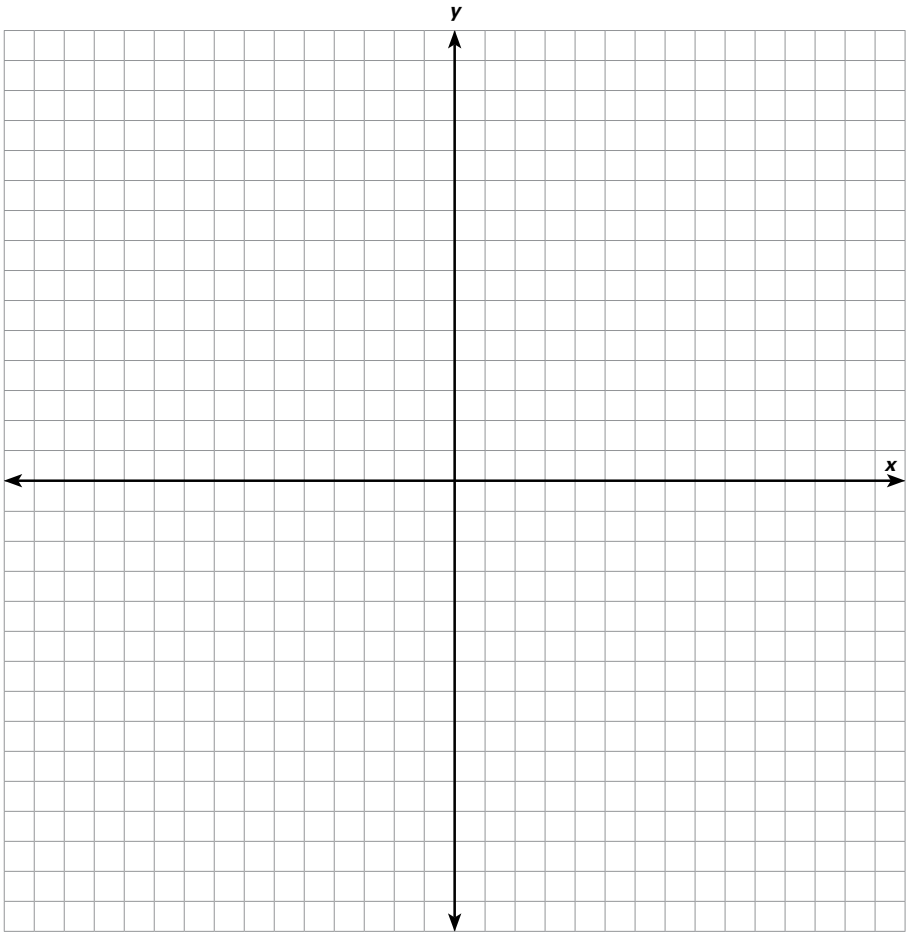
6. Which graph represents a proportional relationship? _____ How do you know?

_____

_____

7. Use the graph for the proportional relationship to complete a **ratio table**.

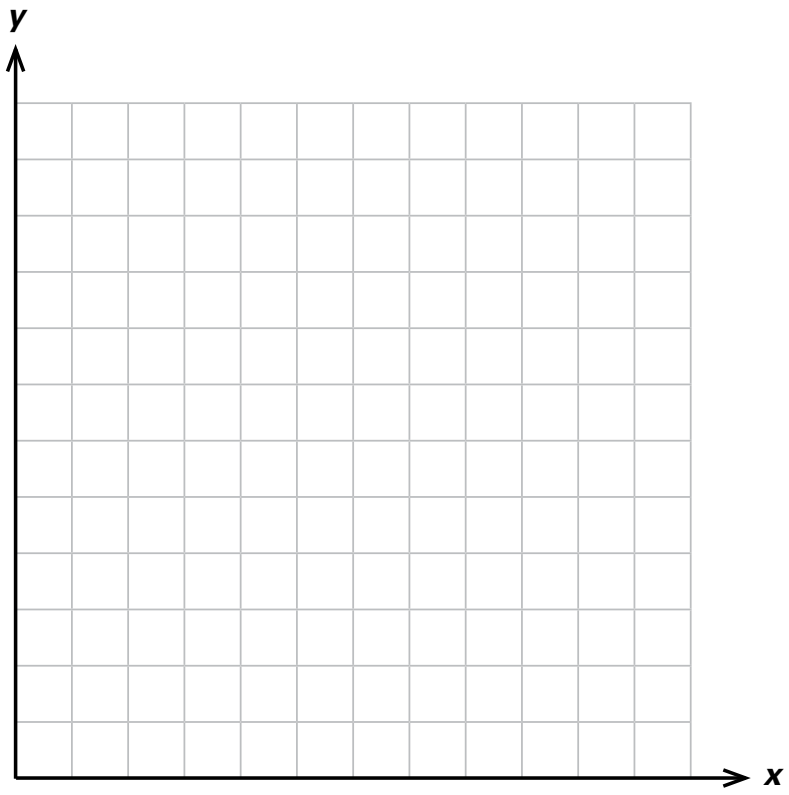
Graph Now



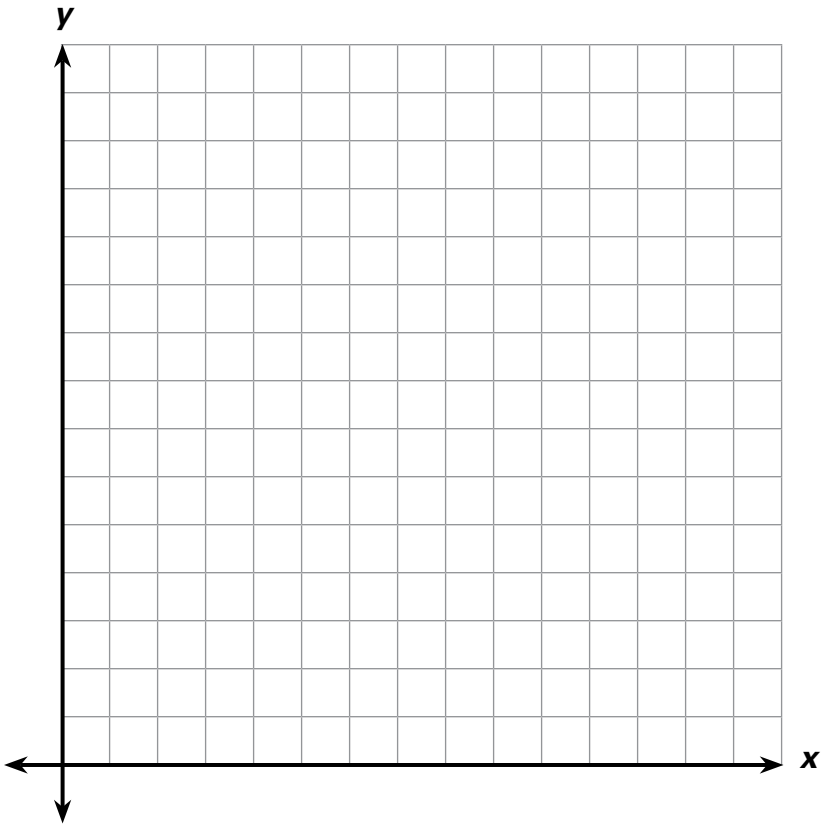
x	*y*
–1	14
4	–10
3	0
5	–5
–12	–11
–7	–9
6	5
–2	15
13	9

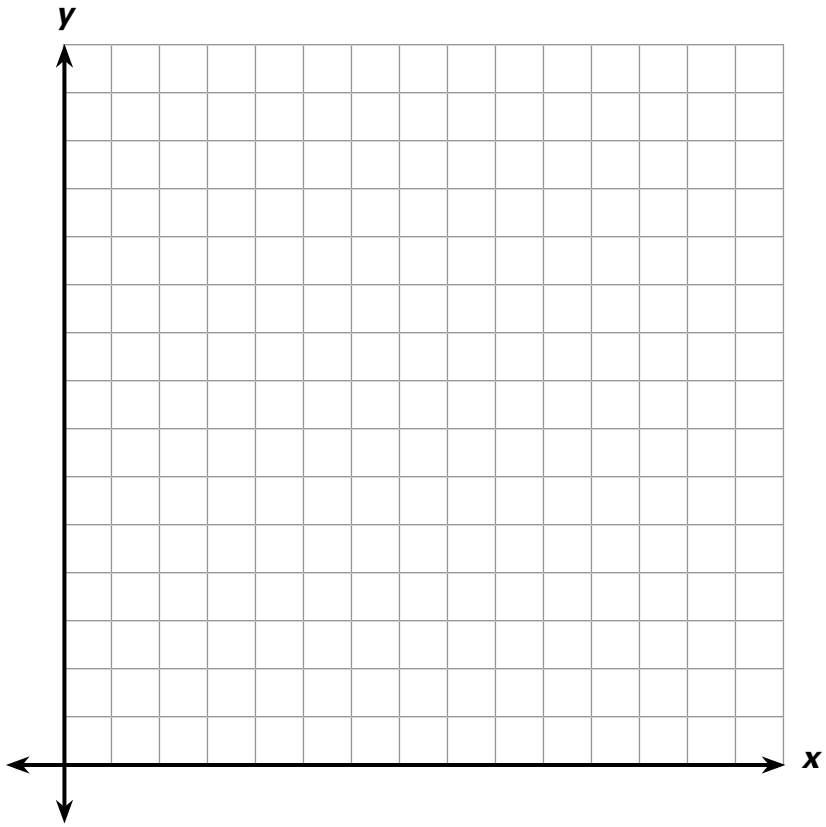
Snowman Buttons

(*x*)	(*y*)

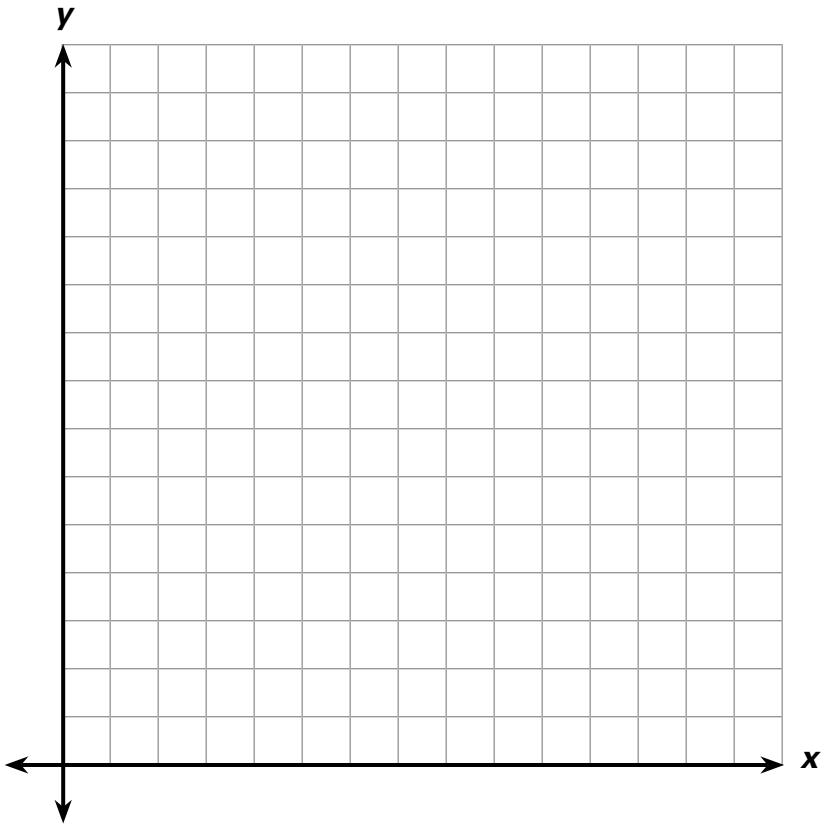


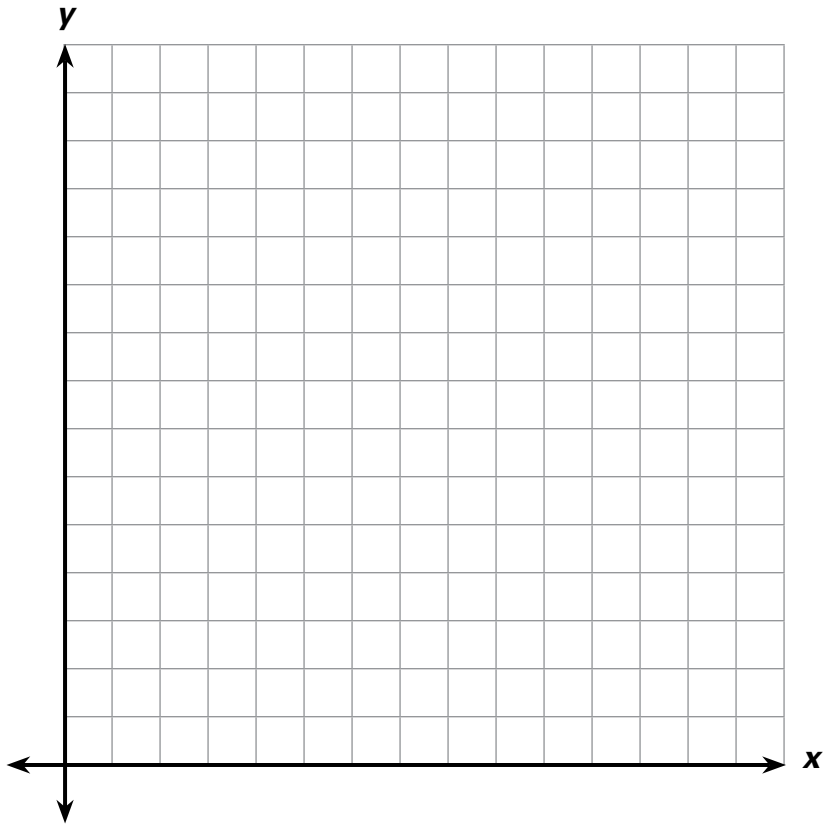
Coordinate Planes (First Quadrant)



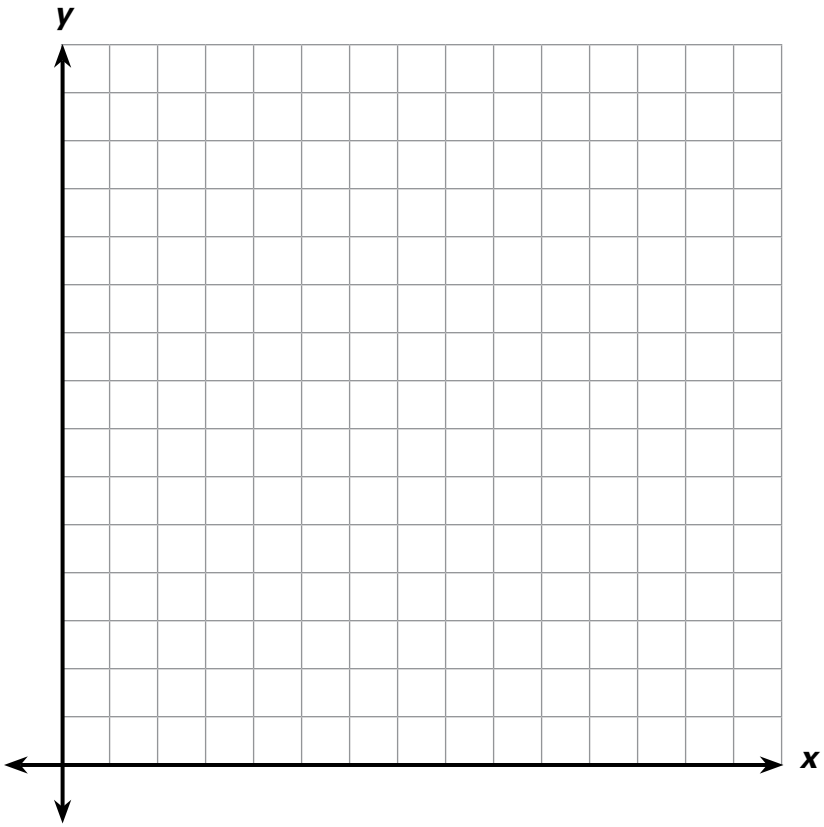


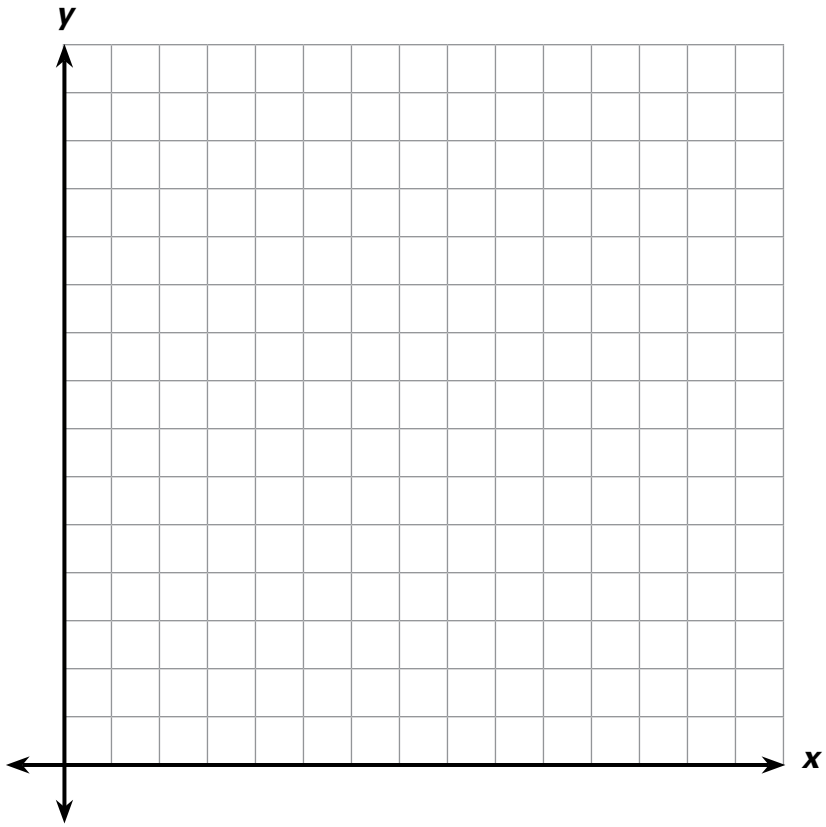
Coordinate Planes (First Quadrant)



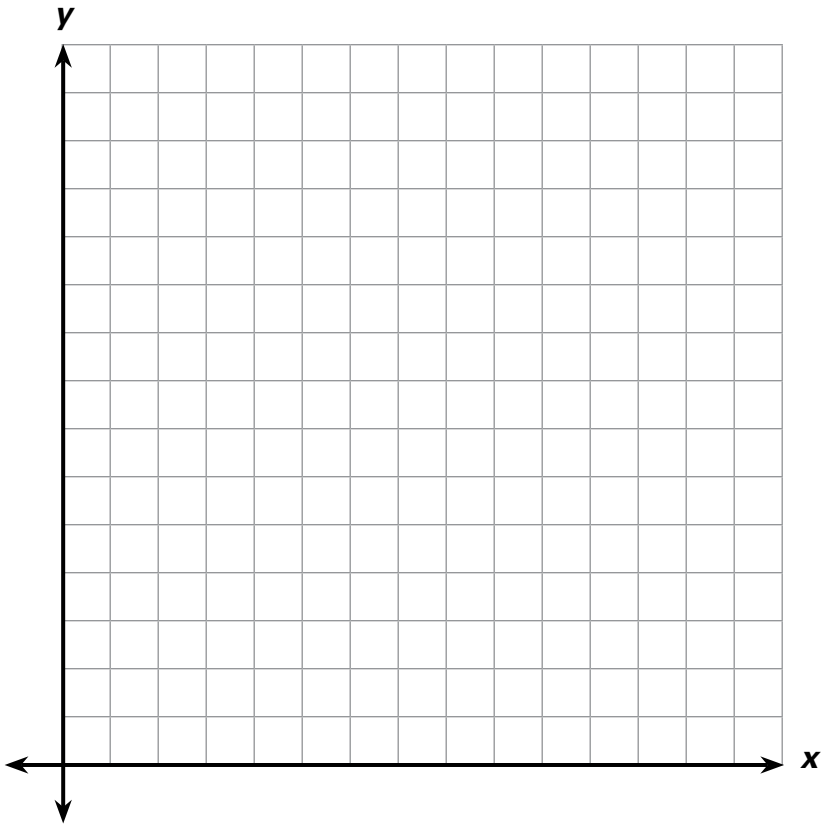


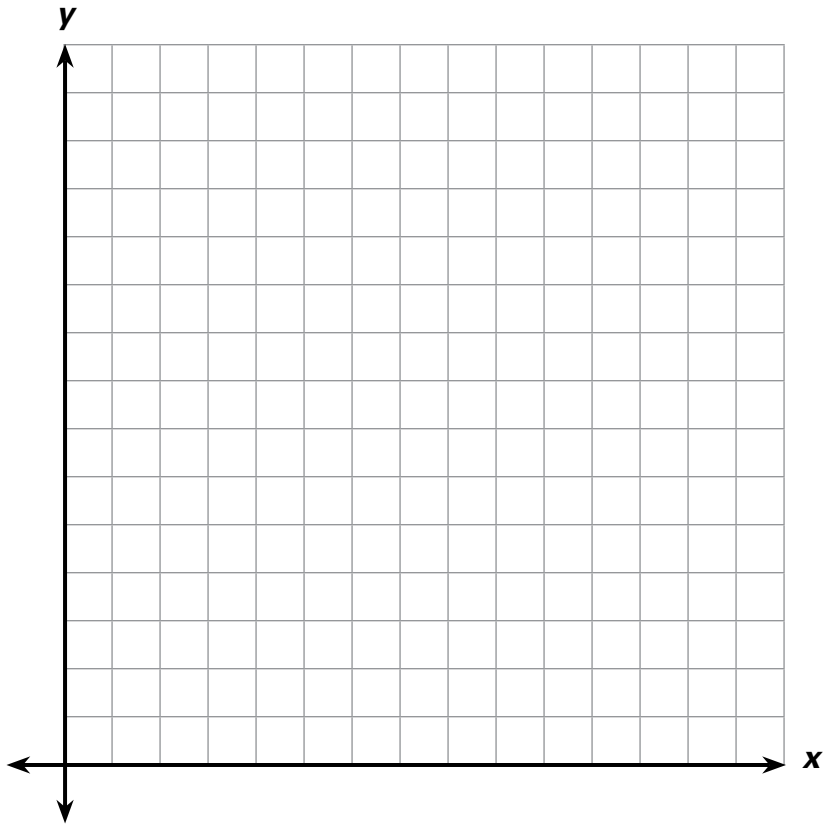
Coordinate Planes (First Quadrant)





Coordinate Planes (First Quadrant)





Screen Printing

Part 1: Solve for *m* in the equation to identify the constant of proportionality.

1. 9 = *m*(3)

Constant of proportionality: _____

2. 11 = *m*(4)

Constant of proportionality: _____

3. 16 = *m*(6)

Constant of proportionality: _____

4. 12 = *m*(3)

Constant of proportionality: _____

5. 8 = *m*(2)

Constant of proportionality: _____

6. 9 = *m*(6)

Constant of proportionality: _____

7. 18 = *m*(8)

Constant of proportionality: _____

8. 13 = *m*(12)

Constant of proportionality: _____

9. 14 = *m*(6)

Constant of proportionality: _____

10. 12 = *m*(6)

Constant of proportionality: _____

Part 2: Identify the dependent and independent variables in the given rate and write an equation to represent the relationship, assuming a constant rate of work. Use *m* to represent the constant of proportionality. Then, solve the equation to identify the constant of proportionality.

11. Khadijah can screen-print 3 T-shirts in 36 minutes.

Equation: _____

Constant of proportionality: _____

12. Khadijah can screen-print 5 posters in 32 minutes.

Equation: _____

Constant of proportionality: _____

13. Khadijah can screen-print 8 stickers in 9 minutes.

Equation: _____

Constant of proportionality: _____

T-shirt Price

Review the example problem. Then use the constant of proportionality and equations to solve.

Example

Khadijah sells **3** screen-printed adult T-shirts for $**24**. What would the price be for **150** T-shirts?

Step 1

Write an equation to represent the relationship in the form of $y = mx$.

$\mathbf{24} = m\mathbf{(3)}$

Step 2

Solve the equation to identify the constant of proportionality.

$\mathbf{24} = m\mathbf{(3)}$

$\frac{\mathbf{24}}{\mathbf{3}} = \frac{m\mathbf{(3)}}{\mathbf{3}}$

$8 = m$

Step 3

Write a new equation to represent the proportional relationship.

y = constant of proportionality(x)

$y = 8x$

Step 4

Substitute **150** for x in the equation and solve.

$y = 8x$
$y = 8\mathbf{(150)}$
$y = 1{,}200$

The price for **150** T-shirts is $1,200.

1. Khadijah also sells children's T-shirts: 3 children's T-shirts are $18. What would the price be for 96 T-shirts? Show your work.

The price of 96 children's T-shirts is $_____.

2. Khadijah sells hats at a rate of 4 hats for $28. What would the price be for 16 hats?

The price of 16 hats is $_____.

3. Khadijah sells socks at a rate of 6 pairs for $3. What would the price be for 21 pairs?

The price of 21 pairs of socks is $_____.

Lesson 26 Exit Ticket

Part 1: Solve the equation to identify the constant of proportionality.

1. $260 = m(4)$

Constant of proportionality: _____

2. $25 = m(20)$

Constant of proportionality: _____

Part 2: Write an equation to represent the relationship. Then use the constant of proportionality to solve the problem.

3. A screen-printing company is printing the programs for a graduation ceremony at the local community college. The company can print 12 programs in 9 minutes.

Equation: _____

Constant of proportionality: _____

How long will it take the company to print all 2,500 programs? Show your work.

It will take the company _____ minutes to print 2,500 programs.

4. On the day of the ceremony, the college asks the company to print certificates for the top 100 graduates. The company can print 8 certificates in 22 minutes.

Equation: _____

Constant of proportionality: _____

How long will it take to print 100 certificates? Show your work.

It will take the company _____ minutes to print 100 certificates.

Extra Practice: Cafeteria Costs

Part 1: Use the ratio to write an equation to represent the relationship. Then, solve the equation to identify the constant of proportionality.

1. 3 cartons of milk cost $2.25.

Equation: _____

Constant of proportionality: _____

2. 8 bananas cost $2.80.

Equation: _____

Constant of proportionality: _____

3. 7 bags of chips cost $5.95.

Equation: _____

Constant of proportionality: _____

4. 5 breakfast burritos cost $12.25.

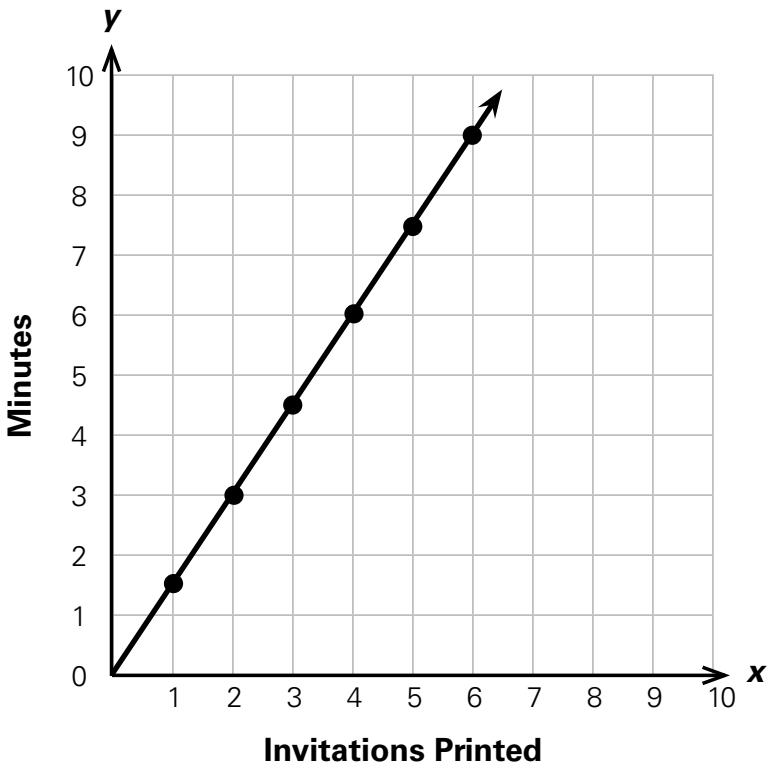
Equation: _____

Constant of proportionality: _____

Part 2: Solve the proportional equations for the given value of x.

$y = \frac{1}{4}x \quad x = 240$ $y =$ _____	$y = 5x \quad x = 225$ $y =$ _____	$y = 9x \quad x = 82$ $y =$ _____
$y = \frac{1}{3}x \quad x = 360$ $y =$ _____	$y = 20x \quad x = 500$ $y =$ _____	$y = \frac{2}{5}x \quad x = 25$ $y =$ _____

Wildlife Rescue Fundraiser



(*x*)	(*y*)	

Concentration Equations

$18 = m(6)$	$10 = m(8)$	$6 = m(3)$
$11 = m(5)$	$10 = m(4)$	$6 = m(1)$
$36 = m(8)$	$7 = m(2)$	$6 = m(5)$
$10 = m(7)$	$11 = m(8)$	$8 = m(3)$

Concentration Constants of Proportionality

$m = 3$	$m = \frac{5}{4}$	$m = 2$
$m = \frac{11}{5}$	$m = \frac{5}{2}$	$m = 6$
$m = \frac{9}{2}$	$m = \frac{7}{2}$	$m = \frac{6}{5}$
$m = \frac{10}{7}$	$m = \frac{11}{8}$	$m = \frac{8}{3}$

Secret Message

Lourdes sent a coded secret message to her friend, Tai, telling her where to meet after school. Where is Tai supposed to meet Lourdes?

Code Key

A	**450**	**N**	**140**
B	**104**	**O**	**600**
C	**2,400**	**P**	**520**
D	**150**	**Q**	**2,250**
E	**20**	**R**	**2,475**
F	**4,500**	**S**	**70**
G	**236**	**T**	**500**
H	**300**	**U**	**425**
I	**1,800**	**V**	**290**
J	**52**	**W**	**700**
K	**203**	**X**	**284**
L	**225**	**Y**	**2,030**
M	**109**	**Z**	**210**

Meet me at the:

_____	_____	_____
$y = 3x$	$y = \frac{3}{2}x$	$y = 29.5x$
for $x = 50$	for $x = 400$	for $x = 8$

_____	_____	_____	_____
$y = 5x$	$y = \frac{3}{4}x$	$y = 11x$	$y = 2.9x$
for $x = 104$	for $x = 600$	for $x = 225$	for $x = 70$

Sailing Away

Part 1: Identify the constant of proportionality in each equation.

1. $d = 12t$

constant of proportionality: _____

2. $b = 11e$

constant of proportionality: _____

3. $s = 10z$

constant of proportionality: _____

4. $m = j$

constant of proportionality: _____

Part 2: Read the problem and answer the questions. Graph the equation on **graph paper**.

5. Tony dreams of sailing a real boat someday. He wonders how long different trips might take. Simon says a good equation for this, assuming a steady cruising rate, is $d = 4.5t$, where d is the distance the boat travels in nautical miles and t is the time in hours.

a. What is the constant of proportionality of the equation? _____

b. Complete the ratio table for the equation.

Time (hours) (t)	**Distance (nautical miles) (d)**
0	
1	
2	
3	
4	
5	

c. Using the ratio table data, write a set of ordered pairs for this relationship.

_____ _____ _____ _____ _____ _____

d. Create a graph of the proportional relationship.

Graph from an Equation

Review the example problem. Then draw a graph to represent the proportional relationship on **graph paper**.

Example

Create a graph for the equation $y = \mathbf{3}x$.

Step 1

Identify the constant of proportionality.

The equation $y = \mathbf{3}x$ is in the form $y = mx$. The value of m, **3**, is the constant of proportionality.

Step 2

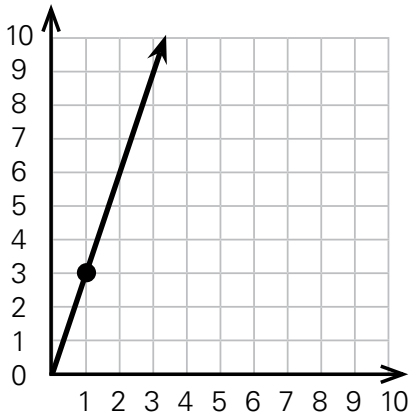
Use the constant of proportionality to find the value of y when $x = 1$.

$y = \mathbf{3}x$ $y = \mathbf{3}(1) = \mathbf{3}$

Step 3

Plot the ordered pair for when $x = 1$. Then draw a line connecting the ordered pair and the origin.

When $x = 1$, $y = \mathbf{3}$, so the ordered pair is (1, **3**).

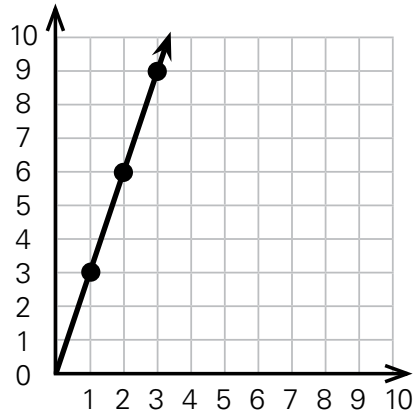


Step 4

Verify that the graph is correct by using the equation to find other ordered pairs that should fall on the line.

$y = \mathbf{3}x$ $y = \mathbf{3}(2) = 6$
ordered pair: (2, 6)

$y = \mathbf{3}x$ $y = \mathbf{3}(3) = 9$
ordered pair: (3, 9)



1. Graph $y = 6x$. Use the equation to list three ordered pairs that fall on the line. Show your work.

2. Graph $y = \frac{1}{2}x$. Use the equation to list three ordered pairs that fall on the line. Show your work.

Lesson 27 Exit Ticket

Part 1: Complete the ratio table to represent the proportional relationship.

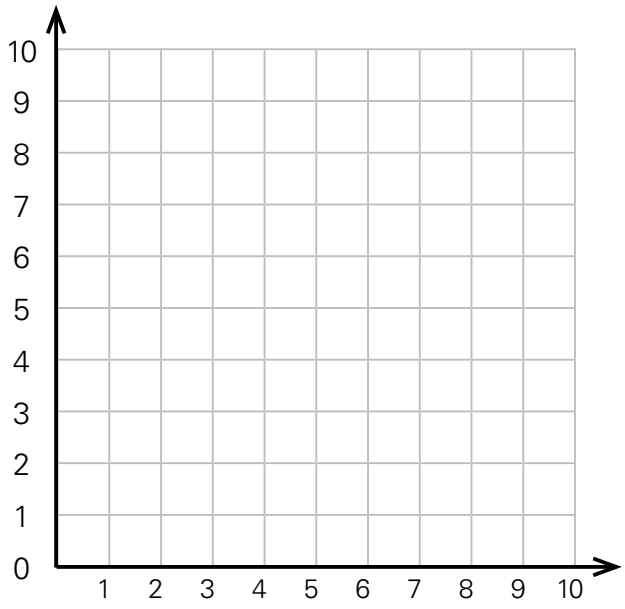
1. Represent the equation $w = 5.25n$, where w is the weight in pounds of n number of powerboat models.

What is the constant of proportionality? _____

Number of Powerboat Models (n)	Weight (pounds) (w)
0	
1	
2	
3	
4	

Part 2: Complete the graph to represent the proportional relationship.

2. Graph $y = 2x$. Use the equation to list three ordered pairs that fall on the line. Show your work.



Ordered pairs: _____ _____ _____

What is the constant of proportionality? _____

Extra Practice: Lunch Time

Part 1: Complete the following statements with *sometimes*, *always*, or *never*.

1. The graph of a proportional relationship _____ goes through the origin.

2. _____ the constant of proportionality is less than one.

3. You can _____ find equivalent ratios by adding the same amount to the numerator as you add to the denominator.

4. The constant of proportionality is _____ equal to 1.

Part 2: Use the constant of proportionality and a ratio table to graph a proportional relationship from an equation in the form $y = mx$ and answer questions.

5. Aunt Emily buys Dylan a meal each time he visits her in the big city. The equation $c = 7n$ represents the cost (c) for a number of meals (n).

a. What is the constant of proportionality for this relationship? _____

b. Complete the ratio table.

Number of Meals (*n*)	Cost (dollars) (*c*)
1	
2	
3	
4	

c. Graph the equation on **graph paper.**

Part 3: Identify the constant of proportionality for each equation.

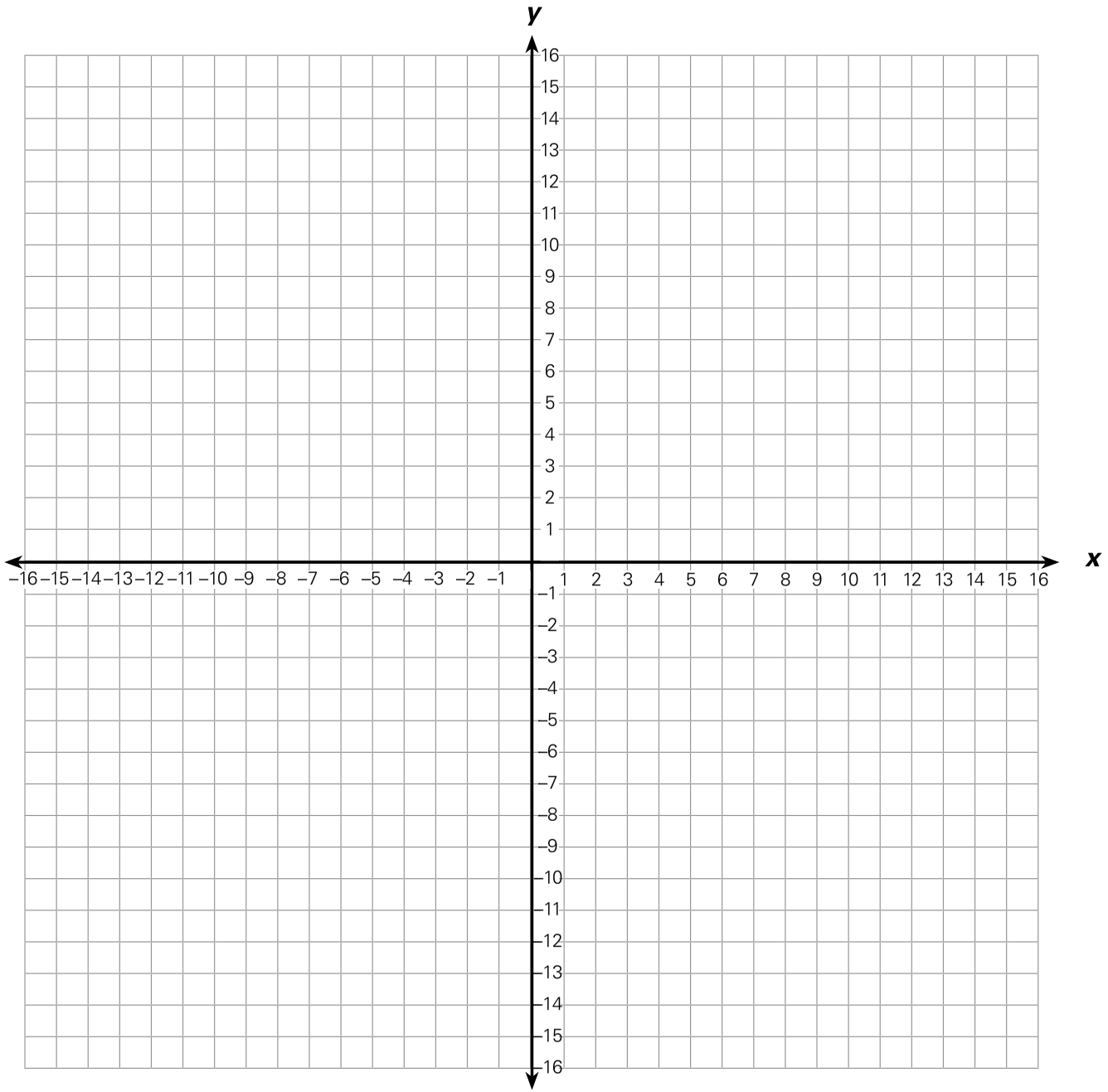
6. $y = 51.2x$ _____

7. $y = x$ _____

8. $y = (\frac{9}{2})x$ _____

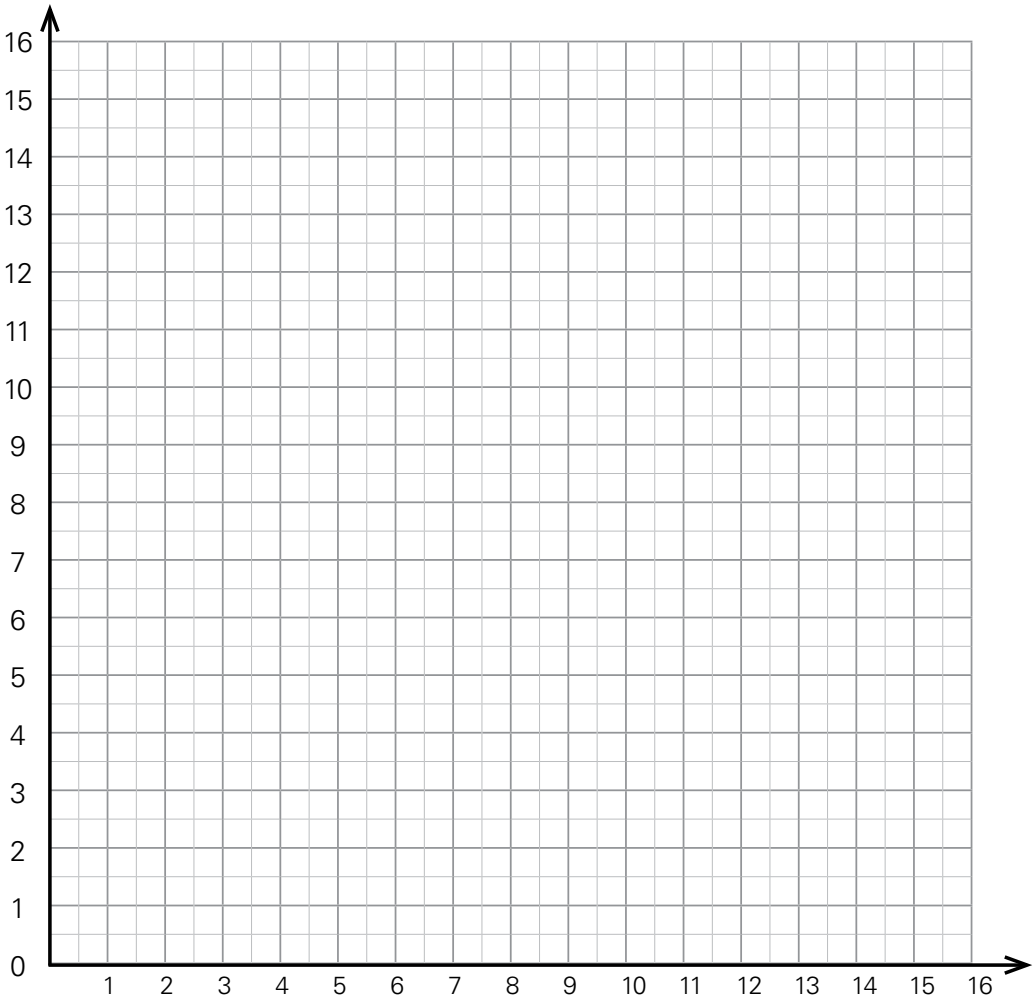
9. $y = 150x$ _____

Graph Now



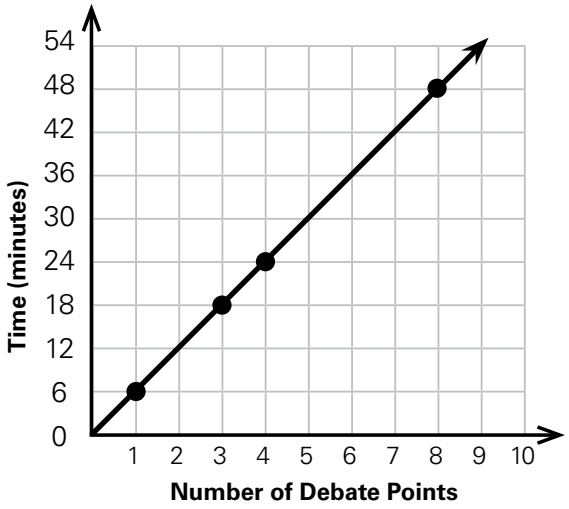
x	y
–8	16
–6	12
–4	8
–1	2
0	0
1	–2
4	–8
6	–12
8	–16

My Sailboat



Tree Debate

Part 1: In the tree tax debate, Zeeshan can make as many points as he wants, but he must use the same amount of time speaking on each point. The graph shows Zeeshan's argument in time spent to points made in favor of the tree tax law. Use the graph to answer the questions.



1. Does the graph show a proportional relationship? Explain.

_____

_____

2. Circle the point that represents the constant of proportionality on the graph.

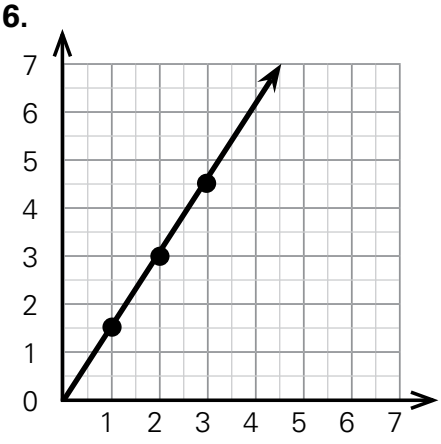
3. What is the constant of proportionality? _____

4. Write an equation to represent the relationship in the form $y = mx$. _____

5. Use the points on the graph and the equation to verify that the constant of proportionality you identified is correct. Show your work.

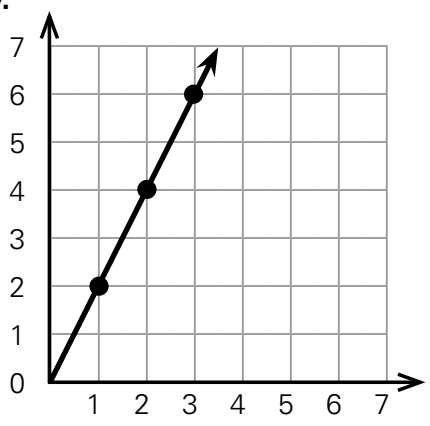
Part 2: Identify the constant of proportionality represented by each graph.

6.



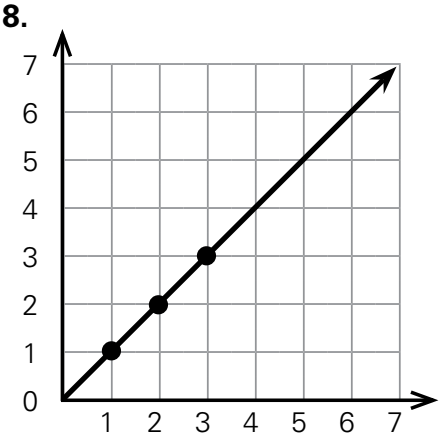
Constant of proportionality: _____

7.



Constant of proportionality: _____

8.



Constant of proportionality: _____

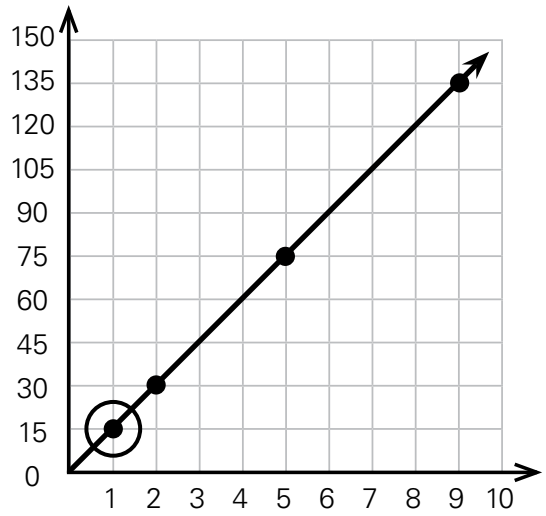
Constant of Proportionality

Review the example problem. Then identify the constant of proportionality and write an equation.

Example

Find the constant of proportionality in the ratio table. Write an equation in the form of $y = mx$.

Step 1



On a ratio table, the constant of proportionality is the value of y when $x = 1$.

x	*y*
0	0
1	**15**
2	30
5	75
9	135

When $x = 1$, $y = 15$, so the constant of proportionality is 15.

Step 2

Write an equation in the form of $y = mx$, substituting the constant of proportionality for m.

$y = mx$ $\quad$ $y = 15x$

1.

x	*y*
0	0
1	11
2	22

Constant of proportionality: _____

Equation: _____

2.

x	*y*
0	0
$\frac{2}{5}$	1
1	$2\frac{2}{5}$

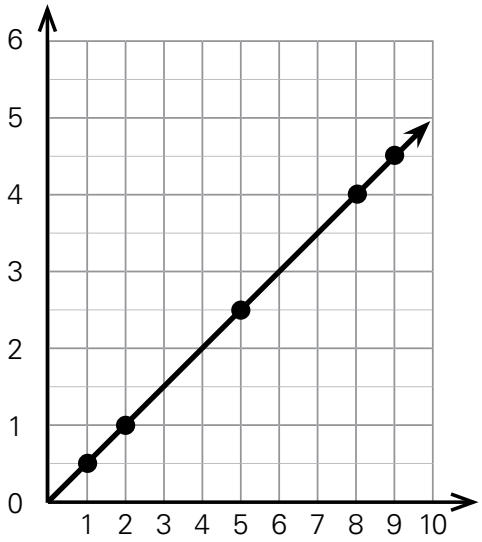
Constant of proportionality: _____

Equation: _____

Lesson 28 Exit Ticket

Identify the constant of proportionality in each graph or ratio table. Then write an equation in the form of $y = mx$ to represent the proportional relationship.

1.



Constant of proportionality: _____

Equation: _____

2.

x	*y*
0	0
1	7
4	28
9	63
15	105
20	140

Constant of proportionality: _____

Equation: _____

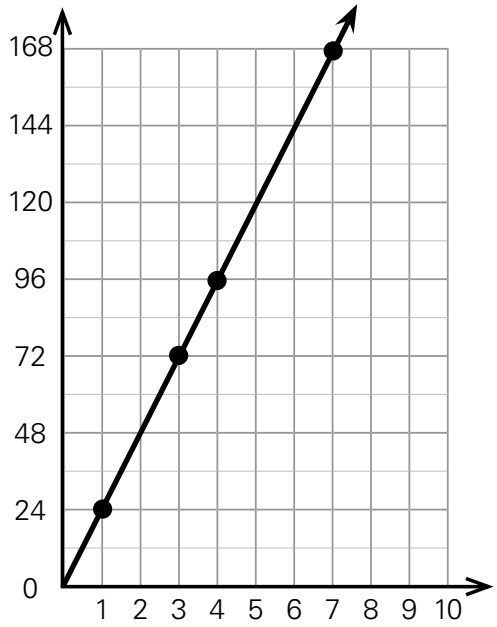
3.

x	*y*
0	0
1	12
3	36
6	72
12	144
30	360

Constant of proportionality: _____

Equation: _____

4.



Constant of proportionality: _____

Equation: _____

Extra Practice: Mosaics

Part 1: Determine whether the following statements are *true* or *false*.

1. The line of a proportional relationship must go through the origin. _____

2. The constant of proportionality is always the value of x when the value of y is 1. _____

3. You can use a graph or a ratio table to write an equation to represent a proportional relationship.

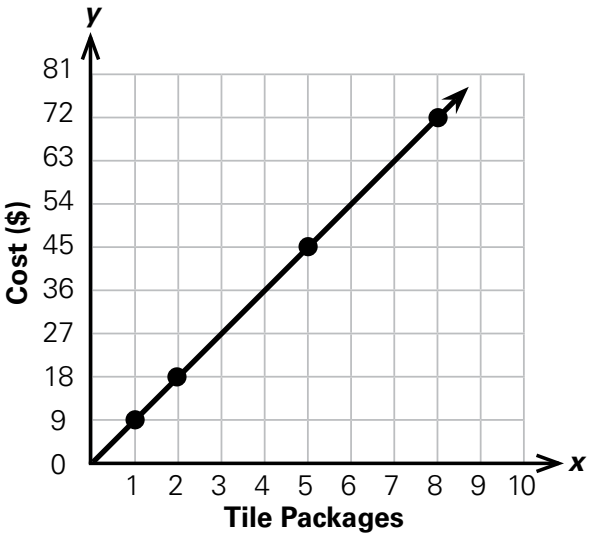
_____

4. You can use points on the graph of a proportional relationship to prove your equation is correct.

_____

5. In the equation $y = mx$, x represents the constant of proportionality _____

Part 2: The graph shows the cost for packages of tiles needed for Isabelle's mosaics. Look at the graph and answer the questions.



6. How can you determine that this relationship is proportional? _____

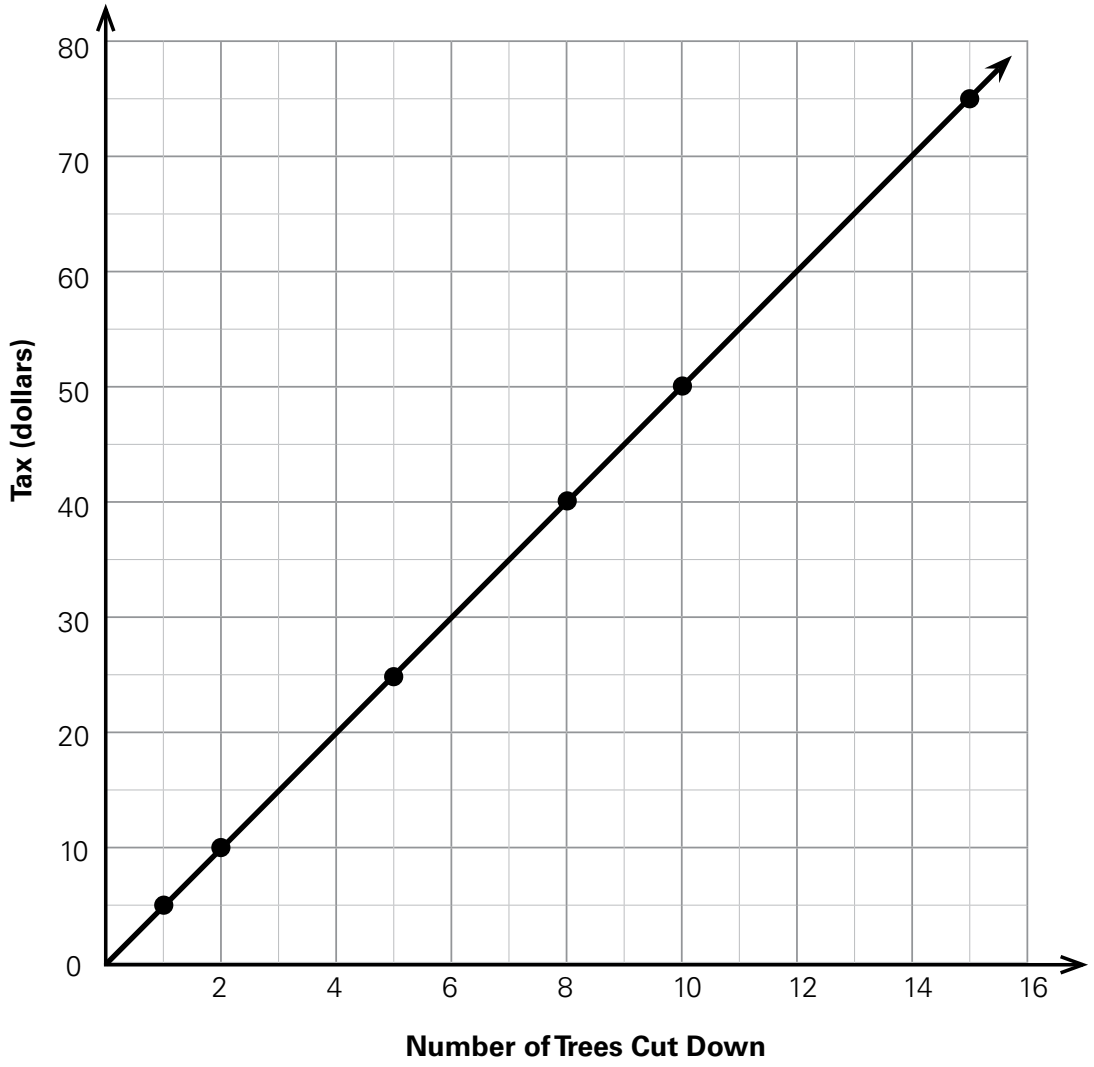
_____

7. What is the constant of proportionality for this relationship? _____

8. Write an equation to represent this relationship. _____

9. Use a point on the line to verify your equation is correct. Show your work.

Tax for Trees Cut Down



Trees Cut Down (x)	Tax (dollars) (y)

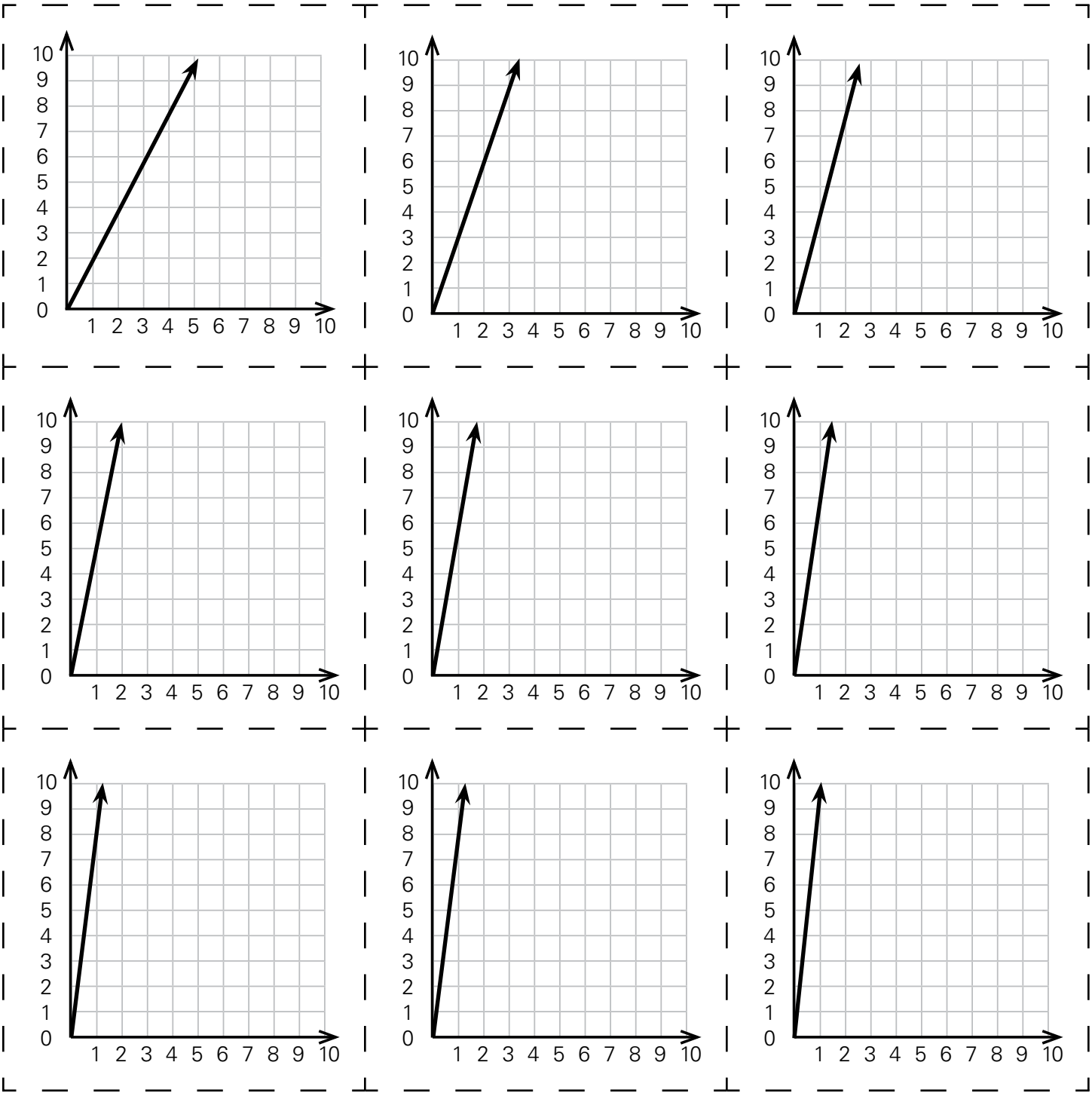
Cost to Plant Trees

Trees Planted (x)	Cost (dollars) (y)
0	0
1	8
3	24
4	32
7	56
9	72

Fast Check

Number of Pizzas (x)	Number of Friends (y)
0	0
$\frac{1}{4}$	1
$\frac{1}{2}$	2
1	4
$\frac{3}{2}$	6
2	8
$\frac{5}{2}$	10
3	12

Equation Sensations Cards



Conveyor Belt

Read the problem. Then answer the questions.

1. The warehouse where Darwin works just installed a shorter conveyor belt to handle small, fragile items. The total length of the new conveyor belt is 175 feet, and a box moves from one end to the other in 5 minutes.

a. Write a ratio to represent the relationship of feet to minutes. _____

b. What is the constant of proportionality of this relationship? Show your work.

_____

The constant of proportionality is _____ feet per minute.

c. Write an equation to represent this relationship. _____

d. Complete the ratio table.

Time (minutes) (*x*)	Distance (feet) (*y*)
0	
1	
2	
3	
4	
5	

e. What are the ordered pairs represented by ratios in the table?

_____ _____ _____ _____ _____ _____

f. On a separate sheet of **graph paper**, graph the ordered pairs.

Box Packing

Review the example problem. Then answer the questions.

Example

At the warehouse, **200** pounds of merchandise fit in **8** boxes. Create an equation, table, and graph to represent this relationship per box.

Step 1

Write a rate of the relationship of pounds to boxes.

200 pounds in **8** boxes = $\frac{\mathbf{200}}{\mathbf{8}}$

Step 2

Divide both parts of the ratio by 8 to identify the unit rate.

$\frac{\mathbf{200}}{\mathbf{8}} \div \frac{8}{8} = \frac{25}{1}$

The unit rate is 25 pounds per box. The constant of proportionality is the value of y in the unit rate, 25.

Step 3

Write an equation in the form $y = mx$, where m = the constant of proportionality.

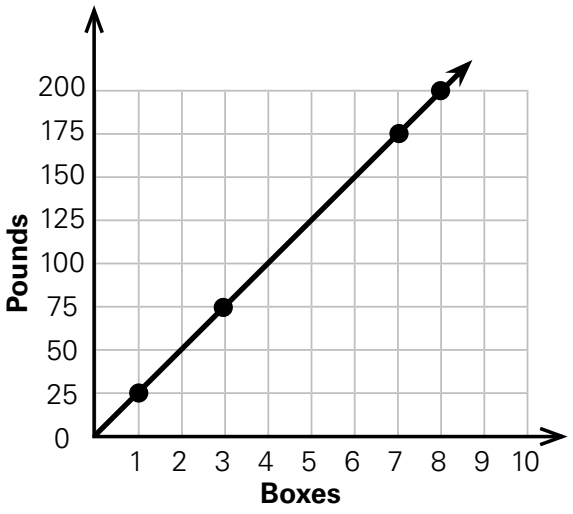
$y = 25x$

Substitute x-values into the equation to find the y-values and complete a ratio table.

Boxes (x)	**Pounds (y)**
0	0
1	25
3	75
7	175
8	**200**

Step 4

Graph ordered pairs found using the equation on a coordinate plane.



1. During an 8-hour shift working at the warehouse, Darwin can pack 96 boxes. Represent the relationship per hour.

What is the constant of proportionality?

_____

What equation represents this relationship?

_____

Graph this relationship on **graph paper**.

2. In 3 hours, workers at the warehouse can pack 48 boxes. Represent the relationship per hour.

What is the constant of proportionality?

_____

What equation represents this relationship?

_____

Draw a **ratio table** for this relationship.

Lesson 29 Exit Ticket

Part 1: Complete the ratio table to represent the proportional relationship. Answer the questions.

1. Darwin has been working really hard at his warehouse job, and his supervisor, Michelle, just gave him a raise! His paycheck for 40 hours is now $800 before taxes are removed. Represent the relationship per hour.

Time (hours) (x)	Salary ($) ($y$)
0	
1	
5	
12	
20	
40	

a. What is the constant of proportionality? Show your work.

_____

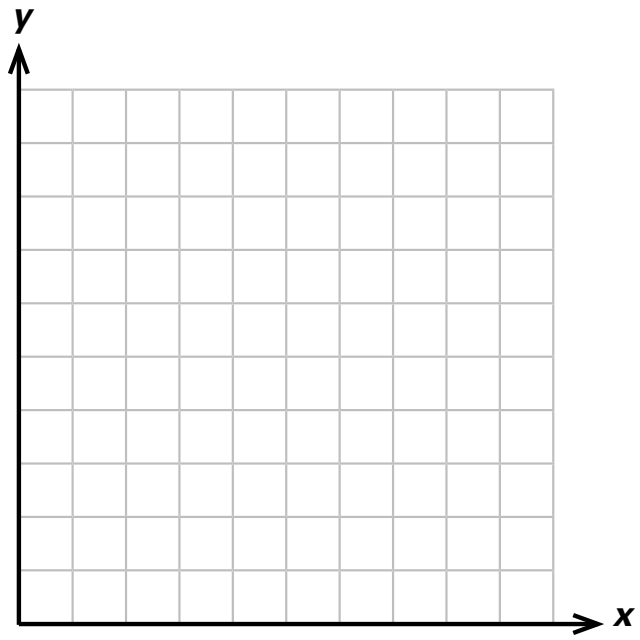
The constant of proportionality is _____.

b. Write an equation to represent this relationship, where y = dollars and x = hours.

_____

Part 2: Complete the coordinate graph to represent the proportional relationship. Answer the questions.

2. A machine at the warehouse packs 36 packages of crackers in 9 boxes. Represent the relationship per box, up to 9 boxes.



a. What is the constant of proportionality? Show your work.

_____

The constant of proportionality is

_____.

b. Write an equation to represent this relationship, where p = packages and b = boxes.

_____

Extra Practice: Trucking

Part 1: A truck that delivers packages from a warehouse drives 300 miles in 5 hours. Use the rate to answer the questions.

1. Write a ratio to represent the relationship of miles to hours. _____

2. What is the constant of proportionality of this relationship? Show your work.

_____

The constant of proportionality is _____

3. The equation _____ represents this relationship.

4. Complete the ratio table.

5. Write the set of ordered pairs from the table.

_____ _____ _____ _____

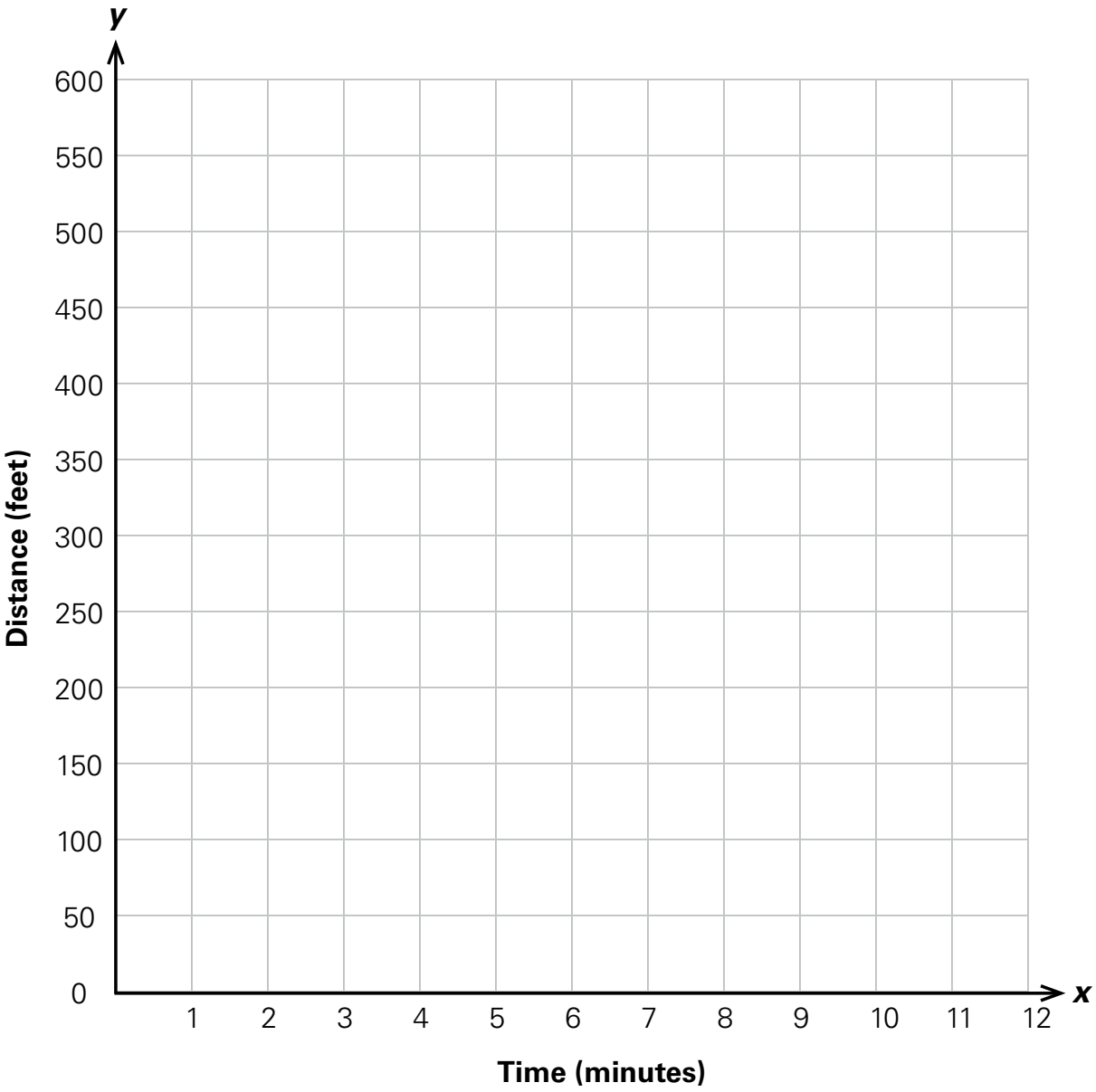
6. Graph the ordered pairs from question 5 on a separate sheet of **graph paper**.

Time (hours) (*x*)	**Distance (miles) (*y*)**
0	
1	
3	
5	

Part 2: Match the relationship with the constant of proportionality.

$39 for 13 gallons of milk	6
54 pairs of socks in 9 packages	11
420 pounds of gravel in 15 buckets	56
396 minutes to knit 36 rows of a scarf	3
1,120 points scored in 20 levels of a video game	28

Conveyor Belt Graph



Ratio Tables

(x)	(y)

(x)	(y)

(x)	(y)

(x)	(y)

Keeping the Beat

Kendra has become very interested in other animals' heart rates. She looked online and saw that an elephant has an average heart rate of 10 beats in 20 seconds. Use this rate to answer the questions.

1. Write a ratio to represent the relationship of heartbeats to seconds. _____

2. Determine the constant of proportionality in beats per minute. Show your work.

The constant of proportionality is _____.

3. Write an equation to represent this relationship using *h* for heartbeats and *t* for time in minutes.

_____

4. Use the equation in question 3 to fill in the ratio table.

5. Look at the row in the table with a star. What do the quantities in that row mean?

_____

_____

Time (minutes) (*t*)	Heartbeats (*h*)
0	
1	
★ 2	
6	
9	

6. On **graph paper**, graph the ordered pairs represented in the ratio table.

7. How many times does an elephant's heart beat in 45 minutes? Use an equation to solve.

An elephant's heart beats _____ times in 45 minutes.

8. On your graph, what does the point at (7, 210) represent?

_____

Interpret Graphs

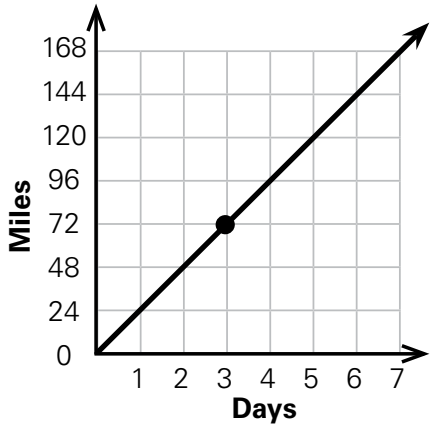
Review the example problem. Then use the graph to answer the questions.

Example

The graph shows how far Randall bicycled. What does the point on the graph mean? Can you use the graph to find out how many total miles Randall biked in 7 days?

Step 1

Identify the coordinates of the given point.



The coordinates are (3, 72).

Step 2

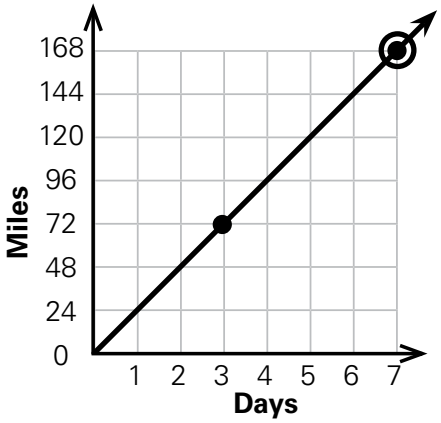
Use the information in the graph to interpret the point.

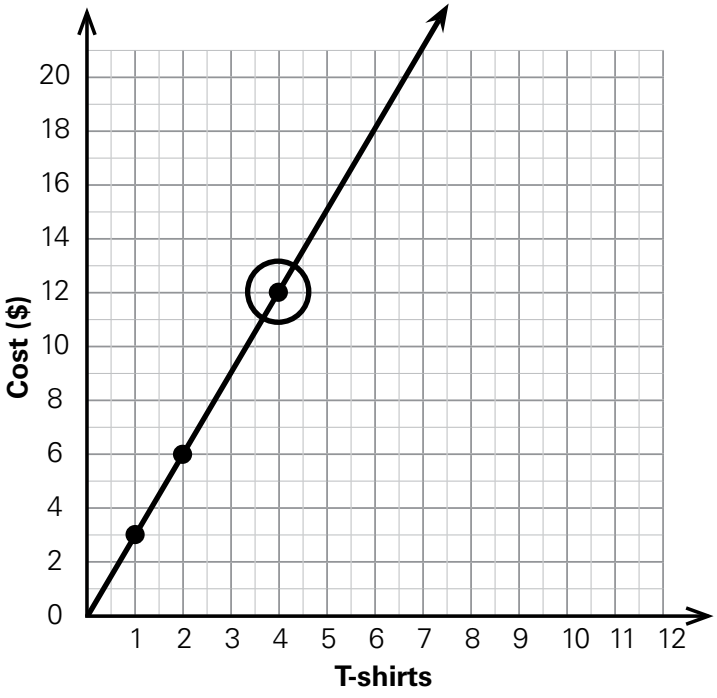
Since x shows the number of days and y shows the number of miles, the point (3, 72) means that Randall biked 72 miles in 3 days.

Step 3

To find how far Randall biked in 7 days, find the point at which $x = 7$.

The point (7, 168) falls on the line, so in 7 days, Randall bikes 168 miles.





Janae was given the graph shown.

1. What does the circled point mean?

_____

2. How much does 1 T-shirt cost? _____

3. How many T-shirts can Janae buy for $6?

_____

4. What equation represents this proportional relationship? _____

5. How much would 21 T-shirts cost? _____

Lesson 30 Exit Ticket

Part 1: Faisan is making vegetable soup. The recipe says that 6 pounds of mixed vegetables (carrots, celery, peas, green beans, corn, and potatoes) will make 42 cups of soup. Use the rate to answer the questions.

1. Write an equation to represent this relationship, using *c* for cups of soup and *p* for pounds of vegetables. Show your work finding the constant of proportionality.

Equation: _____

2. Use the equation in question 3 to complete the ratio table.

Vegetables (pounds) (*p*)	Soup (cups) (*c*)
1	
6	
15	
★ 20	

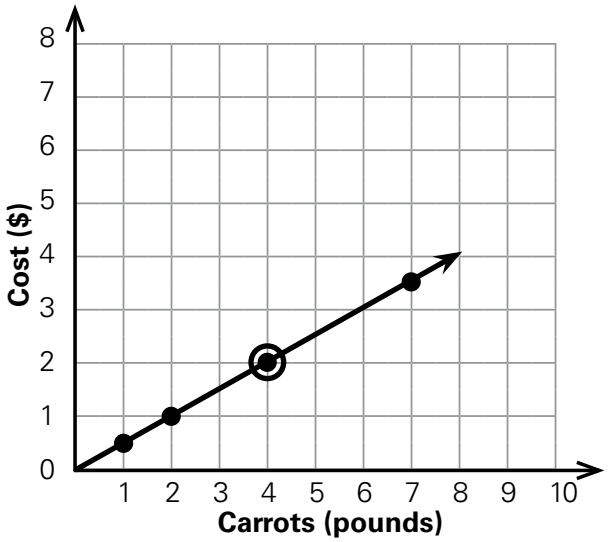
3. Look at the row in the table with a star. What do the quantities in that row mean? _____

_____

4. How many cups of soup will 12 pounds of vegetables make? Show your work.

12 pounds of vegetables will make _____ cups of soup.

Part 2: The graph shows the cost of different amounts of carrots at the grocery store. Use the graph to answer the questions.



5. What does the circled point mean?

_____

6. How much does 7 pounds cost? _____

7. How many pounds can you buy for $3?

_____

8. What equation represents this proportional relationship?_____

9. How much would 10 pounds cost? _____

Extra Practice: Shampoo Sale

Kendra saw two ads for the dog shampoo she uses on her dog Blossom. Use the ads to answer the questions.

1. Explain how Kendra can use representations of proportional relationships to determine which store has a better deal on Blossom's shampoo.

2. Write equations for each store's sale, where z is the number of ounces and d is dollars.

Pet Palace: _____

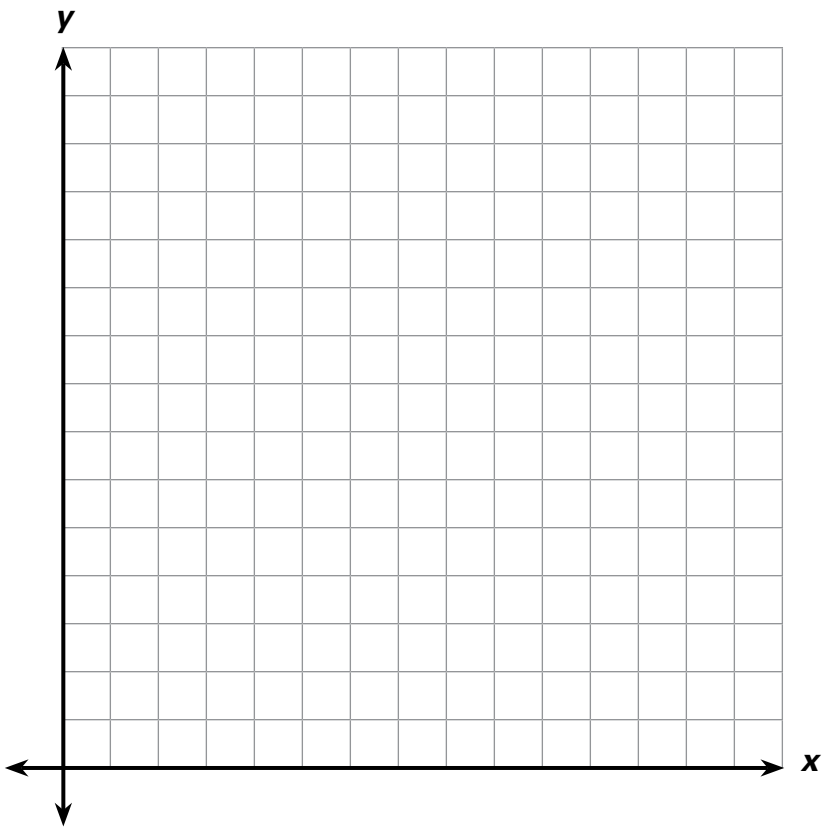
Bow-Wow Boutique: _____

3. On a separate sheet of **graph paper**, create a graph to represent each store's price.

4. Which store's sale gives Kendra more ounces of Blossom's shampoo per dollar?

Blossom's Heartbeats

(*x*)	(*y*)



Assessment

Unit 4 Assessment

1. Nadya is making quarts of fruit punch from a can of powder mix. She needs to determine how many scoops of powder to use for different quarts of punch. The can says to use 3 scoops of powder for 4 quarts of water.

a. What is the constant of proportionality? Show your work.

The constant of proportionality is _____.

b. Complete the ratio table using the constant of proportionality.

Water (quarts) (*x*)	**Powder Mix (scoops) (*y*)**
0	0
1	
2	
3	
4	
5	

2. Identify the constant of proportionality in each ratio table below.

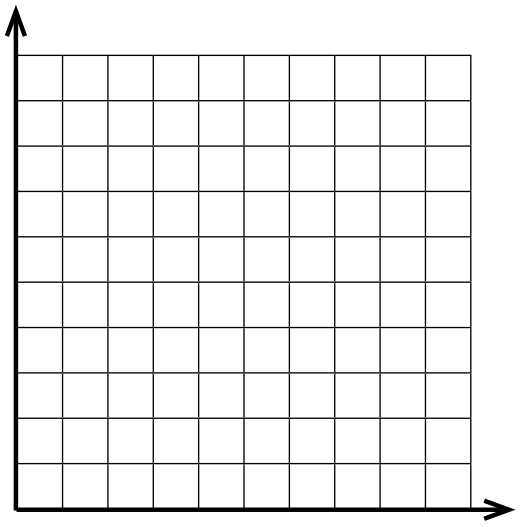
Milk (ounces) (*x*)	Cocoa (teaspoons) (*y*)
0	0
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

The constant of proportionality is _____.

Kitten's Weight (pounds) (*x*)	Feeding Amount (calories) (*y*)
0	0
1	60
2	120
3	180

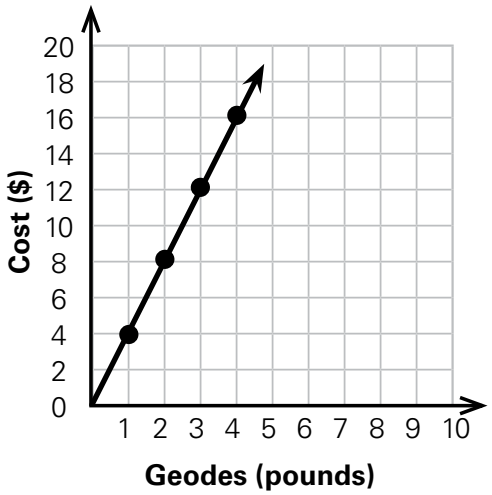
The constant of proportionality is _____.

3. The price of a box of crayons with 8 crayons is 80 cents. Draw a graph to show the price per crayon.



What is the constant of proportionality? _____

4. Doug likes to go hiking to collect rocks. At a special place, he can collect geodes with sparkly crystals inside, but he has to pay for the geodes he collects. The graph shows how much Doug has to pay for different weights of geodes.



a. What is the constant of proportionality? _____

b. Complete the ratio table for the pounds of geodes to the cost.

Geodes (pounds)	Cost ($)
0	
1	
2	
3	
4	

5. It takes David's baby sister 7.5 seconds to walk 3 feet.

a. What is the constant of proportionality for the relationship of seconds per feet? Show your work.

The constant of proportionality is _____.

b. How long will it take David's sister to walk 8 feet?

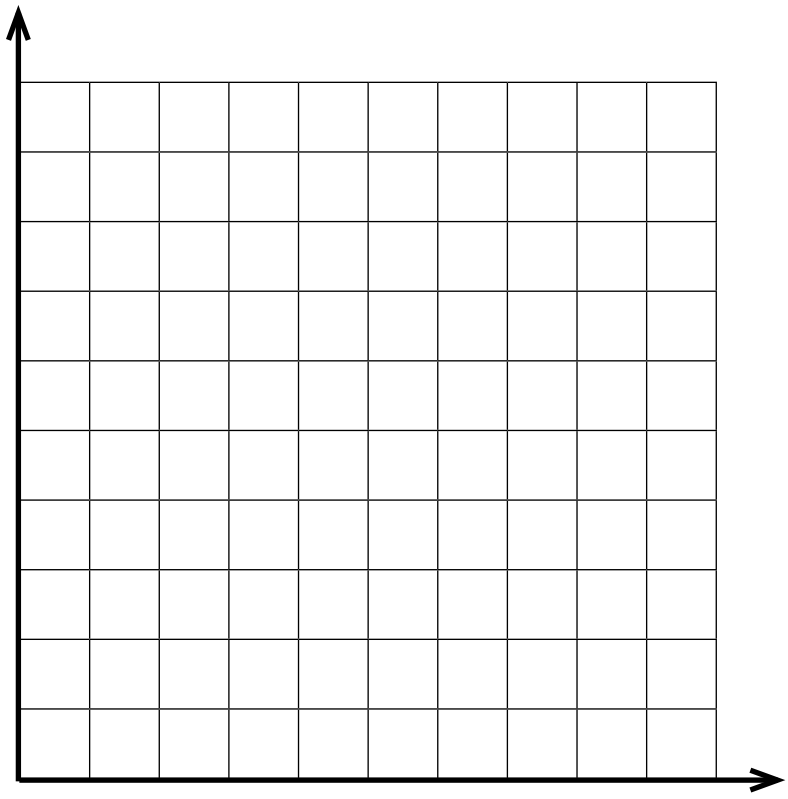
It will take her _____ seconds to walk 8 feet.

6. Vanessa is training for a marathon. She runs every day and keeps a log of her running. The equation $d = 9t$ represents the distance d (in miles) for the time t (in days).

a. Complete the ratio table to show the proportional relationship.

Time (days) (t)	**Distance (miles) (d)**
0	0
1	
2	
3	
4	

b. Graph the equation $d = 9t$.



7. The ratio table shows the points Caleb scored in the levels as he progressed through his video game.

Levels (x)	Points (y)
0	0
1	45
3	135
12	540
25	1,125

a. What is the constant of proportionality? _____

b. Write an equation to represent this relationship. _____

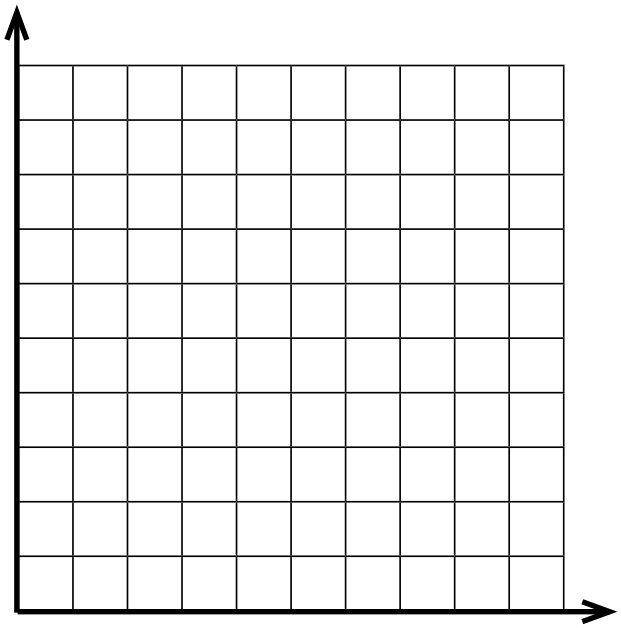
8. Hiro has a dog-walking business after school. He gets paid for each dog he walks. Last month he made $108 for walking 9 dogs. Represent the relationship per dog.

a. What is the constant of proportionality for this relationship? Show your work.

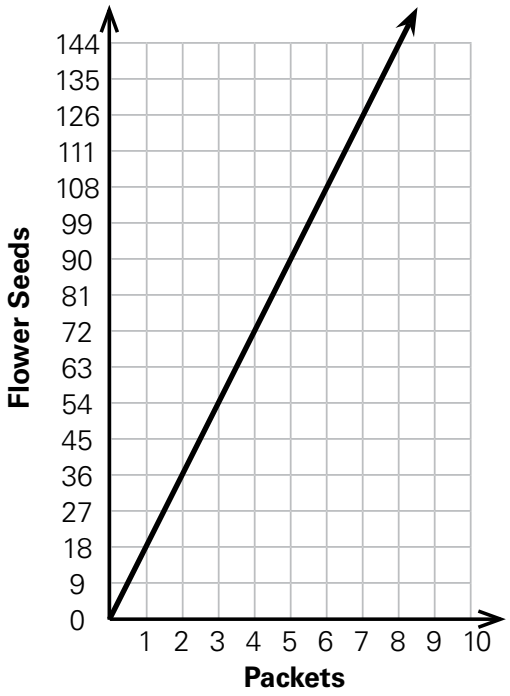
The constant of proportionality is _____.

b. Write an equation to represent this relationship. _____

c. Graph the proportional relationship.



9. The graph shows the number of flower seeds in different numbers of packets.



a. What does the point (4, 72) represent? _____

b. How many flower seeds are in 7 seed packets? _____

10. Aaron's blueberry scone recipe calls for 3 cups of blueberries to make 48 scones.

a. Write an equation in the form $y = mx$ to represent this relationship, using b for cups of blueberries and s for scones. _____

b. How many scones will 8 cups of blueberries make? Show your work.

8 cups of blueberries will make _____ scones.

Unit 4 Cumulative Review

1. Keenan stains 3 yards of the family deck in 4 minutes.

a. Draw a tape diagram to model Keenan's rate.

b. His brother Zachary stains 6 yards in 8 minutes. Extend your tape diagram to show whether Zachary's rate is proportional to Keenan's rate. Are their rates proportional? _____

Explain how you know using each brother's rate._____

_____

_____

_____

2. Juliana is a photographer. Of the photos she takes, 17 out of 25 are in color. The others are in black and white.

a. What is the ratio of color photos Juliana took? _____

b. What percent of Juliana's photos are in color? _____

3. Divide.

$-36 \div 9 =$ _____

4. Aleksi started his backpacking trip at the top of a canyon. On the first day, he hiked down 4 miles into the canyon and camped. On the second day, he hiked down another 11 miles. Write an addition equation and a subtraction equation using negative integers to model how far down Aleksi hiked by the end of the second day.

Addition: _____

Subtraction: _____

5. Simplify using PEMDAS. Show your work.

$17 + (48 \div 8)^2 - 22 =$ _____

6. Madison rolls her wheelchair 2 miles in 136 minutes. Write an equation to represent the relationship. Then find the constant of proportionality.

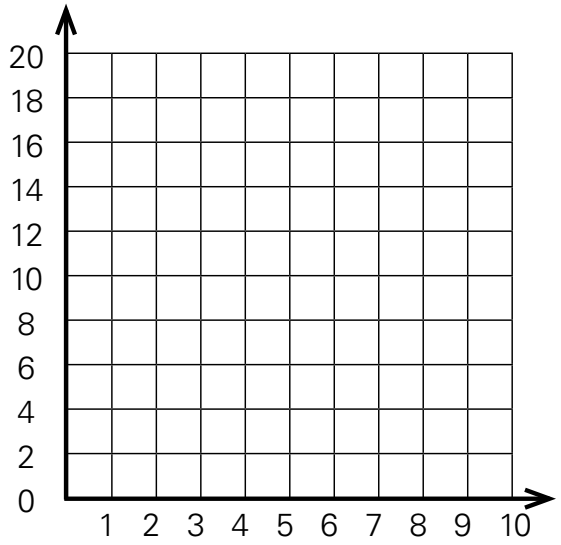
Miles	Time (minutes)
0	0
1	68
2	136
3	204
4	272

Equation: _____

Constant of proportionality: _____

7. Graph the equivalent ratios in the table below on the coordinate plane.

Time (hours)	Salary ($)
0	0
1	4
2	8
3	12
4	16
5	20



8. Multiply.

$15 \times -7 =$ _____

9. There are 120 cans of green beans in 5 boxes. Write an equation in the form $y = mx$ to represent this relationship. _____

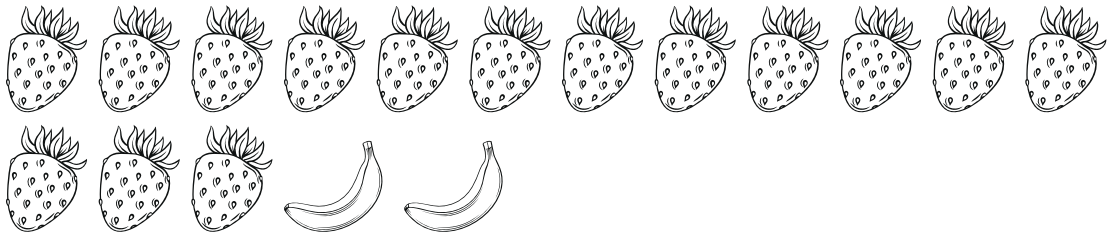
10. Solve for p. Show your work.

$34 + p = 100$

$p =$ _____

11. What is the unit rate per pound for pineapples if they cost $5.70 per 3 pounds? _____

12. To make a fruit smoothie, Akeelah uses the fruit shown.

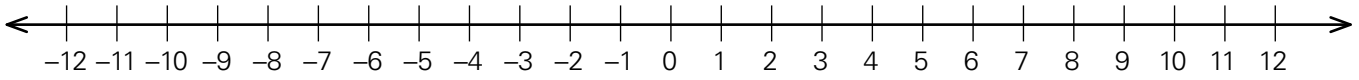


What is the ratio of strawberries to bananas? _____

13. Subtract.

–41 – 19 = _____

14. Plot $w > -6$ on the number line.



15. Kiana ran a 15-mile race in 5 hours. What is Kiana's unit rate?

Kiana's unit rate is _____ miles per hour.

Unit 5:

Equivalent Expressions

Catapult Learning™

Band Banquet

Draw **tape diagrams** to add or subtract the expressions in each problem.

1. Cade is helping with the food for the banquet. Each table will have a basket of the same number of rolls. 11 tables will have Italian rolls, and 7 tables will have whole-wheat rolls. Also, each table will have 2 gluten-free rolls.

Let r = rolls that are not gluten-free. The expressions for rolls on each type of table are:

Italian roll tables: $11r + 22$ and **Whole-wheat roll tables:** $7r + 14$

a. Draw tape diagrams on a separate page to show how to add the expressions.

b. Write a combined expression as the sum of the expressions for the different types of tables. Use the properties of operations to simplify the expression. Show your work.

2. The band is hoping to raise about $400. Danika is trying to figure out how close they will be if she sells 10 tickets and the rest of the band sells 20 total. She doesn't know yet how much the band will sell each ticket for. She does know that a supporter of the band will donate $3 for each ticket sold.

Let m = money raised per ticket. The expressions for the amounts raised are:

Danika selling 10 tickets: $10m + 30$ and **Everybody else selling 20 tickets:** $20m + 60$

a. Draw a tape diagram on a separate page to show the amounts raised as parts of the total $400 that needs to be raised.

b. Write a new expression that shows the difference between 400 and the sum of the expressions. Use the properties of operations to simplify the expression. Show your work.

Adding and Subtracting Expressions

Review the example problem. Then use operation properties to simplify expressions with one variable using addition and/or subtraction. Show your work.

Example

Subtract the expression $\mathbf{4}\boldsymbol{b} + \mathbf{6}$ from the expression $\mathbf{5}\boldsymbol{b} + \mathbf{15}$. Simplify. Write an equation.

Step 1

If subtracting, treat the subtraction sign as a negative sign and distribute.

$(\mathbf{5}\boldsymbol{b} + \mathbf{15}) - (\mathbf{4}\boldsymbol{b} + \mathbf{6})$
$(\mathbf{5}\boldsymbol{b} + \mathbf{15}) + (-1 \times \mathbf{4}\boldsymbol{b}) + (-1 \times \mathbf{6})$
$(\mathbf{5}\boldsymbol{b} + \mathbf{15}) + (-\mathbf{4}\boldsymbol{b}) + (-\mathbf{6})$

Step 2

Use the associative property to group like terms.

$(\mathbf{5}\boldsymbol{b} + \mathbf{15}) + (-\mathbf{4}\boldsymbol{b}) + (-\mathbf{6}) = \mathbf{5}\boldsymbol{b} - \mathbf{4}\boldsymbol{b} + \mathbf{15} - \mathbf{6}$

Step 3

Simplify and write an equation.

$(\mathbf{5}\boldsymbol{b} - \mathbf{4}\boldsymbol{b}) + (\mathbf{15} - \mathbf{6}) = b + 9$
$(\mathbf{5}\boldsymbol{b} + \mathbf{15}) - (\mathbf{4}\boldsymbol{b} + \mathbf{6}) = b + 9$

1. Add.
$(3x + 12) + (24 + 12x) =$ _____

2. Subtract.
$(27 + 23a) - (13a + 6) =$ _____

3. Subtract.
$250 - (10c + 9) - (8c + 9) =$ _____

4. Add.
$400 + (5b + 13 + 2b + 7 + 16b + 32) =$
_____

5. Subtract.
$(22d + 6) - (3d + 13) =$ _____

6. Simplify.
$10e + 11 + 12e + 13 + 14e + 15 - 16e - 17 =$
_____

Lesson 32 Exit Ticket

Part 1: Draw tape diagrams and use operation properties to add or subtract the expressions in each problem.

1. $(7p + 4) - (3p + 2) =$ _____

2. $(5t + 5) + (7t + 8) =$ _____

Part 2: Use operation properties to simplify expressions using addition and/or subtraction. Show your work.

3. Subtract.
$(27k + 12) - (10k + 6) - (4k + 1) =$

_____

4. Add.
$(7d + 34) + (15d + 9) + (2d + 46) =$

_____

Extra Practice: Band Car Wash

Part 1: The Imperial Middle School marching band is having a car wash fundraiser. The students are working in two shifts, and they took reservations ahead of time. The money raised will come from washing cars and from cash donations that were also promised ahead of time. The scheduled car washes and expected donations are shown in the table.

Shift 1	Shift 2
10 car washes	12 car washes
$48 in donations	$75 in donations

Let p = the price per car wash. The expressions for the money expected from the two shifts are:

Shift 1: $10p + 48$ and **Shift 2:** $12p + 75$

1. Draw a tape diagram to simplify the expression.

2. Write a combined expression as the sum of the expressions for the money from the two shifts. Use the properties of operations to simplify the expression. Show your work.

Part 2: Simplify the expressions.

3. $(42a + 16) + (18a + 32) + (5a + 29) =$ _____

4. $(100n + 48) - (51n + 14) =$ _____

5. $(33j + 8) + (9j + 36) =$ _____

6. $(245g + 84) - (3g + 11) - (82g + 26) =$ _____

7. $(56x + 7) + (8x + 21) + (32x + 12) =$ _____

8. $(80b + 65) - (42b + 33) =$ _____

9. $(126c + 72) + (48c + 15) =$ _____

Adopt-a-Thon

Read the problem. Use **tape diagrams** and the distributive property of multiplication to factor the expression and answer questions.

1. On Sunday, the Adopt-a-Thon was even more successful than on Saturday. This time, cats were more popular than dogs. There were 4 times as many cats as dogs adopted. Each dog adopter received 4 coupons for dog food. Each cat adopter received 6 coupons for cat food.

Let d = the number of dogs adopted. The expressions for the coupons given out are:

Coupons to dog adopters: $4d$ and **Coupons to cat adopters:** $24d$

The combined expression for the total number of coupons given out on Sunday is $4d + 24d$.

a. Draw a tape diagram on a separate sheet to model each term in the expression.

b. Use the greatest common factor to divide each tape into equal groups. Circle the groups.

c. Write the factored expression for the number of coupons given out on Sunday.

---

2. Adopt-a-Thon volunteers gave away 3 squeaky toys per dog that was adopted. They also gave away 1 squeaky toy to each of the first 12 people who came to the event.

Let d = the number of dogs adopted. The expressions for the squeaky toys given out are:

Squeaky toys to dog adopters: $3d$ and **Squeaky toys to first 12 people:** 12

The combined expression for the total number of squeaky toys given out is $3d + 12$.

a. Draw a tape diagram on a separate sheet to model each term in the expression.

b. Use the greatest common factor to divide each tape into equal groups. Divide the whole-number tape into the number of sections equal to the GCF.

c. Write the factored expression for the number of squeaky toys given out on Sunday.

Factoring Expressions

Review the example problem. Then identify the greatest common factor (GCF) for the terms in each expression. Use the GCF and the distributive property of multiplication to factor the expressions.

Example

Find the greatest common factor for the terms in the expression. Factor the expression **–10*x*** + **15x**.

Step 1	Step 2	Step 3
Identify the terms in the expression. The terms are **–10*x*** and **15x**.	Find the GCF for the expression terms. • Both terms include *x*. • Common factors include 1*x* and 5*x*. • The GCF is 5*x*.	Use the distributive property of multiplication to factor the expression. 5*x* (–2 + 3)

1. 25*a* + (–30*a*) GCF: _____ Factored expression: _____

2. 16b – 8*b* GCF: _____ Factored expression: _____

3. –12*c* + (–27*c*) GCF: _____ Factored expression: _____

4. 4*d* + 16 GCF: _____ Factored expression: _____

5. 34 + 17*e* GCF: _____ Factored expression: _____

6. *f* + 100*f* GCF: _____ Factored expression: _____

Lesson 33 Exit Ticket

Part 1: Draw tape diagrams and use the distributive property of multiplication to factor expressions.

1. $4p + 12p$

a. Draw a tape diagram to model each term in the expression. Use the greatest common factor to divide each tape into equal groups. Circle the groups.

b. Write the factored expression. _____

2. $10q + 8q$

a. Draw a tape diagram to model each term in the expression. Use the greatest common factor to divide each tape into equal groups. Circle the groups.

b. Write the factored expression. _____

Part 2: Identify the greatest common factor for the terms in each expression. Use the GCF and the distributive property of multiplication to factor the expressions.

3. $-7y + -35y$ GCF: _____ Factored expression: _____

4. $100t + 48$ GCF: _____ Factored expression: _____

Extra Practice: More Factored Expressions

Part 1: Determine whether the following statements are *true* or *false.*

1. The greatest common factor of two expression terms is any shared factor of the terms.

_____

2. An expression and its factored form are equivalent expressions. _____

3. You can only find the GCF of an expression by using a factor tree. _____

4. In a factored expression, one term will always equal 1. _____

Part 2: Match each expression with its factored expression.

5.

$27x + 18x$	$12x(3 + 2)$
$36x + 24x$	$8x(1 + 7)$
$12x + 48x$	$16x(2 + 5)$
$32x + 80x$	$20x(2 + 3)$
$40x + 60x$	$12x(1 + 4)$
$8x + 56x$	$9x(3 + 2)$

Part 3: Identify the GCF and factor each expression.

6. $15a + 60a$ GCF: _____ Factored expression: _____

7. $42j + 12j$ GCF: _____ Factored expression: _____

8. $225r + 25r$ GCF: _____ Factored expression: _____

9. $21c + 33c$ GCF: _____ Factored expression: _____

Expression Factoring Match Game

Expression

$5x + 50$

Factored Expression

$5(x + 10)$

Expression

$50 - 5x$

Factored Expression

$5(10 - x)$

Expression

$30x + 30$

Factored Expression

$30(x + 1)$

Expression

$30x + 50x$

Factored Expression

$10x(3 + 5)$

Expression Factoring Match Game

Expression

$3x + 4x$

Factored Expression

$x(3 + 4)$

Expression

$9x + 8x$

Factored Expression

$x(9 + 8)$

Expression

$8x - 12x$

Factored Expression

$4x(2 - 3)$

Expression

$15x - 20x$

Factored Expression

$5x(3 - 4)$

Park Pieces

Part 1: Expand the expressions using the distributive property of multiplication.

1. Breanna notices that the architects have changed their designs for a park. The area of one part of the park is now $5(40x + 30x)$. Another part's area is now $4(60x + 40x)$.

a. $5(40x + 30x) =$ _____

b. $4(60x + 40x) =$ _____

2. $-12(2x + 12x) =$ _____

3. $\frac{3}{4}(4t - 16t) =$ _____

4. $36b(10 + 2 + 4) =$ _____

Part 2: Test if the expressions are equivalent by using the distributive property of multiplication to expand the first expression.

5. $11a(3 + 9 + 12)$ and $33a + 99a + 144a$

a. Expand $11a(3 + 9 + 12)$.

b. Are the expressions equivalent? _____

6. $-20b(3 + 4 - 5)$ and $-60b + (-80b) + 100b$

a. Expand $-20b(3 + 4 - 5)$.

b. Are the expressions equivalent? _____

Test Equivalency

Review the example problem. Use the distributive property of multiplication to test whether the expressions are equivalent within each equation. Show your work expanding the first expression within each equation.

Example

Does **$6(8x + 13x + 5x) = 48x + 78x + 30x$**?

Step 1

Multiply the factor by the first term in the first expression to start expanding the expression.

$6(8x + 13x + 5x)$
$(6)(8x)$
$48x$

Step 2

Multiply the factor by the second term.

$6(8x + 13x + 5x)$
$(6)(13x)$
$78x$

Step 3

Multiply the factor by the third term.

$6(8x + 13x + 5x)$
$(6)(5x)$
$30x$

Step 4

Combine the terms of the first expression and compare the two expressions.

$6(8x + 13x + 5x) = 48x + 78x + 30x$

Yes, the expressions are equivalent.

1. Does $15(4f + 20f) = 60f + 300f$?

_____

2. Does $-4(4x + 25x) = -16x + 100$?

_____

3. Does $8(-3a + 7a + 12a) = -24a - 56a - 96a$?

_____

4. Does $4b(3 + 40) = 12 + 160b$?

_____

5. Does $\frac{1}{5}(10c + 30c) = 2c + 6c$?

_____

6. Does $-3d\,(22 - 15 - 5) = -66d + 45d + 15d$?

_____

7. Does $7e(7 + \frac{1}{7} + 11) = 49e + e + 77e$?

_____

8. Does $f\,(1 - 1 + 1 - 3) = f - f - f + 3f$?

_____

Lesson 34 Exit Ticket

Part 1: Use the distributive property of multiplication to expand each expression.

1. $10(-3x + 12x + 25x) =$ _____

2. $2(28a - 42a) =$ _____

3. $6(5p + 8p + 15p) =$ _____

4. $3m(14 - 4 + 25) =$ _____

Part 2: Use the distributive property of multiplication to expand the first expression within each equation and test whether the expressions in the equation are equivalent.

5. Does $-20x(5 - 20) = -100x - 400$? _____

6. Does $\frac{1}{2}(72x + 36 - 16) = 36x + 18 - 8$? _____

Extra Practice: Expanded Expressions

Part 1: Complete the following statements with *sometimes, always,* or *never.*

1. Two expressions _____ represent the same value.

2. You can _____ use the distributive property of multiplication to expand a factored expression.

3. The width of an area model is _____ a variable.

Part 2: Match each expression with its corresponding expanded expression.

4.

$2(6x + 5x + 8x)$	$16x + 36x + 48x$
$4(4x + 9x + 12x)$	$24x + 12x + 18x$
$9(4x + 5x + 3x)$	$12x + 10x + 16x$
$3(8x + 3x + 4x)$	$24x + 9x + 12x$
$2(12x + 6x + 9x)$	$36x + 45x + 27x$

Part 3: Use the distributive property of multiplication to expand each expression.

5. $7(11q + 26q + 7q)$ _____

6. $21(3k + 12k)$ _____

7. $18(8n + 9n + 5n)$ _____

8. $9(33g + 6g)$ _____

Tile Patterns

Ms. Clyburn asked other students in her class to come up with expressions they think are equivalent to Walker's expression of $4n + 4$. Draw tape diagrams and use properties of operations to test for equivalent expressions and answer the questions.

1. A'ja came up with the expression $3(n + 1) + (n + 1)$. Draw a tape diagram to simplify A'ja's expression.

Is A'ja's expression equivalent to Walker's expression? _____

2. Mohammed came up with the expression $4(n + 2) – 2$. Draw a tape diagram to simplify Mohammed's expression.

Is Mohammed's expression equivalent to Walker's expression? _____

3. Mia came up with the expression $2(n + 1)^2 + n^2$. Draw a tape diagram to simplify Mia's expression.

Is Mia's expression equivalent to Walker's expression? _____

Is It Equivalent?

Review the example problem. Use properties of operations to test whether expressions are equivalent and determine if the given equations are true. Show your work.

Example

Are the expressions **2(3*x* + 2) + 4*x*** and **3(2*x* + 3) + 4*x*** equivalent?

Step 1

Expand the first expression using the distributive property.

2(3*x* + 2) + **4*x***
(**2**)(**3*x***) + (**2**)(**2**) + **4*x***
6*x* + 4 + **4*x***

Step 2

Combine like terms and simplify the first expression.

6*x* + 4 + **4*x***
6*x* + **4*x*** + 4
10*x* + 4

Step 3

Use properties and combine like terms to simplify the second expression if needed.

3(2*x* + 3) + 4*x*
(**3**)(**2*x***) + (**3**)(**3**) + **4*x***
6*x* + 9 + **4*x***
10*x* + 9

Step 4

Compare the two simplified expressions.

10*x* + 4 and 10*x* + 9

The expressions are not equivalent.

1. $3(k^2 + 3) - 8 = 3k^2 + 1$

Is the equation true? _____

2. $(k + 3)(3k + 2) = 3k^2 + 1$

Is the equation true? _____

3. $a + a^2 - 3a(a + 3) = 2a^2 + 8a$

Is the equation true? _____

4. $-b(a + 3b) + 3b = -ba$

Is the equation true? _____

5. $c^2(3a + 1) - 1 = 3c^2a + c^2 - 1$

Is the equation true? _____

6. $d^2 + d(3d + 1 - 2a) + a = 4d^2 + d - 2ad + a$

Is the equation true? _____

7. $4(x^2 + 3x) - 12 = 4x^2$

Is the equation true? _____

8. $-2(x^2 + 3y - 5) - 12 = -2x^2 - 6y - 2$

Is the equation true? _____

Lesson 35 Exit Ticket

Part 1: Draw tape diagrams and use properties of operations to test whether expressions are equivalent and answer questions.

1. $3(4d + 2)$ and $2(2d + 3) + 2(4d)$

a. Draw a tape diagram to simplify $3(4d + 2)$.

b. Draw a tape diagram to simplify $2(2d + 3) + 2(4d)$.

c. Are the expressions equivalent? _____

Part 2: Use properties of operations to test whether expressions are equivalent and determine if the given equations are true. Show your work.

2. $5(3x)^2 - (6x + 10x^2) = x(20x - 6) + 15x^2$

a. Simplify the first expression.

b. Simplify the second expression.

c. Is the equation true? _____

3. $3(4c + 5) + 7(2c)^2 = (14c + 1)(2c - 1) + 16$

a. Simplify the first expression.

b. Simplify the second expression.

c. Is the equation true? _____

Extra Practice: Equivalent Expressions

Part 1: Use the tape diagram and properties of operations to explain what Kendra did in each row.

Kendra wanted to see if the expression $4(t + 2) - 5 + 3t$ is equivalent to $7t + 3$. She drew the tape diagram as shown to simplify $4(t + 2) - 5 + 3t$. Explain what Kendra did at each step.

$t + 2$	$t + 2$	$t + 2$	$t + 2$	-5	$3t$	
$t + t + t + t + 3t$			$2 + 2 + 2 + 2 + (-5)$			
$7t$			3			

Part 2: Match each expression with its equivalent expression.

$18m + 36$	$m(27 + m) - 18m + m^2$
$5(9m + 1) + 2(2)$	$9(2m + 4)$
$2m^2 + 9m$	$23m + 18$
$3(6 + 9m) - 4m$	$6m(m + 3) - 9(m + 2)$
$6m^2 + 9m - 18$	$45m + 9$

Part 3: Circle the expressions that are equivalent to $24x + 16$. Draw a line through the expressions that are not equivalent.

$4(6x + 7) - 12$

$2(2x + 8) + 5(4x)$

$(4x)(x + 4) + (24 - 4x^2)$

$3(8x + 6)$

$5(6x + 3) - (6x - 1)$

$8(3x + 2)$

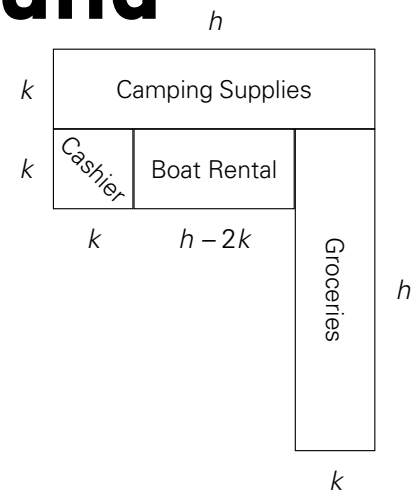
$2(10x + 8)$

$4(6x + 2)$

$5(4x - 2) + (4x + 6)$

Funville Campground

Use the tape diagram and properties of operations to find equivalent expressions and answer questions.



1. Tyrell's dad has told Tyrell that he can organize the sections of the campground store. Tyrell came up with this design and this expression for the store's area: $hk + hk + k(h - 2k) + k^2$.

a. Complete the tape diagram to find an equivalent expression to represent the store area.

A				

b. When you combine all like terms, what is the simplified equivalent expression to represent the area of the store?

_____

2. Tyrell's sister Janeese came up with an expression for the area of the tent camping section of the campground: $rp + (r - p)(r - 3p)$

a. Complete the tape diagram to find an equivalent expression to represent the tent camping area.

A				

b. When you combine all like terms, what is the simplified equivalent expression to represent the tent camping area?

_____

Find Equivalent Expressions

Review the example problem. Then use properties of operations to find equivalent expressions for the given expressions.

Example

Find an equivalent expression for $\boldsymbol{ab + a(b - a) + ab - ba}$.

Step 1	Step 2	Step 3
Identify any factored terms and use the distributive property of multiplication to expand the terms. The term $\boldsymbol{a(b - a)}$ expands to $ab - a^2$.	Group any like terms. Note that terms with the same variables in a different order are like terms—and that you can change the order of the terms. Also, coefficients do not affect like-term groupings. $\boldsymbol{ab +} ab - a^2 \boldsymbol{+ ab - ba}$ $\boldsymbol{ab +} ab \boldsymbol{+ ab - ba} - a^2$ $\boldsymbol{ab +} ab \boldsymbol{+ ab - ab} - a^2$	Combine like terms. Make sure to only combine terms that include the exact same variables and variable exponents. $ab + ab + ab - ab - a^2$ $2ab - a^2$ This is an equivalent expression.

1. $x(y - x) + x(y - x) + x(y - x)$

Equivalent expression: _____

2. $(m + n)(m + 2n) - n^2$

Equivalent expression: _____

3. $-3j(k + j) - k^2$

Equivalent expression: _____

4. $13 - b + 3a(5a + 7b) - a + 57$

Equivalent expression: _____

Lesson 36 Exit Ticket

Part 1: Use a tape diagram and properties of operations to find an equivalent expression.

1. $c^2 + c(d - c) + c(d - c) + c^2$

a. Draw a tape diagram.

b. What is an equivalent expression? _____

2. $x^2 + xy - x(y - x) + 2xy$

a. Draw a tape diagram.

b. What is an equivalent expression? _____

Part 2: Use properties of operations to find equivalent expressions.

3. $4ab + b(a + b) - a^2 =$ _____

4. $g(f - g) + f(g - f) =$ _____

Extra Practice: Expressions Everywhere

Part 1: Determine whether the following statements are *true* or *false.*

1. In a tape diagram modeling an expression, each term should be in its own section. _____

2. If you can determine the side lengths of a multi-section shape, you can write an expression for the area of the shape. _____

3. Each expression has only one equivalent expression. _____

Part 2: List as many equivalent expressions as you can for the given expression.

4. $3xy - x^2 - x^2 + 4x(y - x) + y^2 + 5y(y - x)$

Part 3: Use grouping like terms and properties of operations to find an equivalent expression.

5. $mn(m + n)$ _____

6. $(a + b)(a - b)$ _____

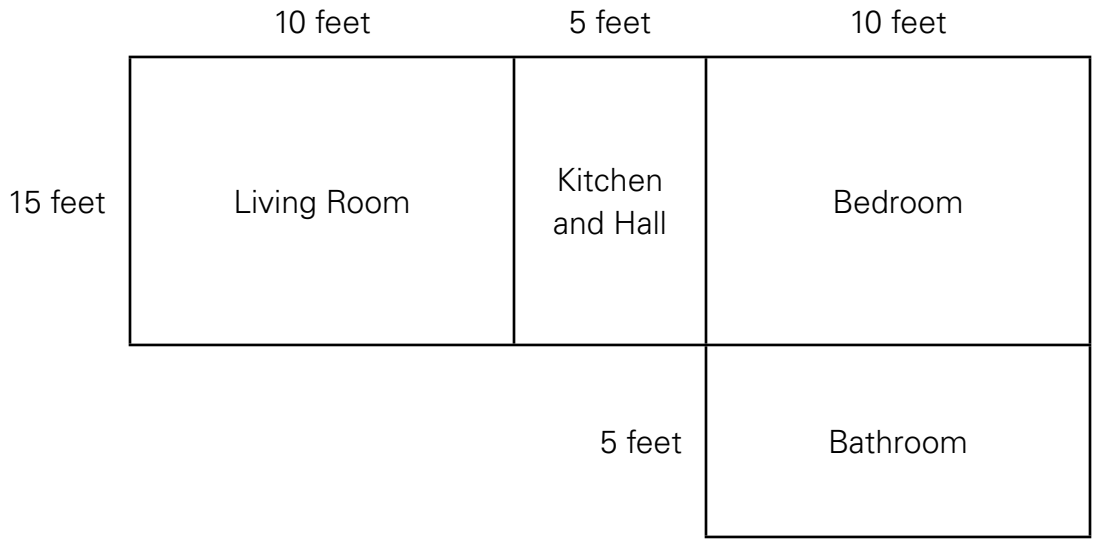
7. $d(c + d) + dd - cc$ _____

8. $g^2 + h(g - h) - h^2 + g(g - h)$ _____

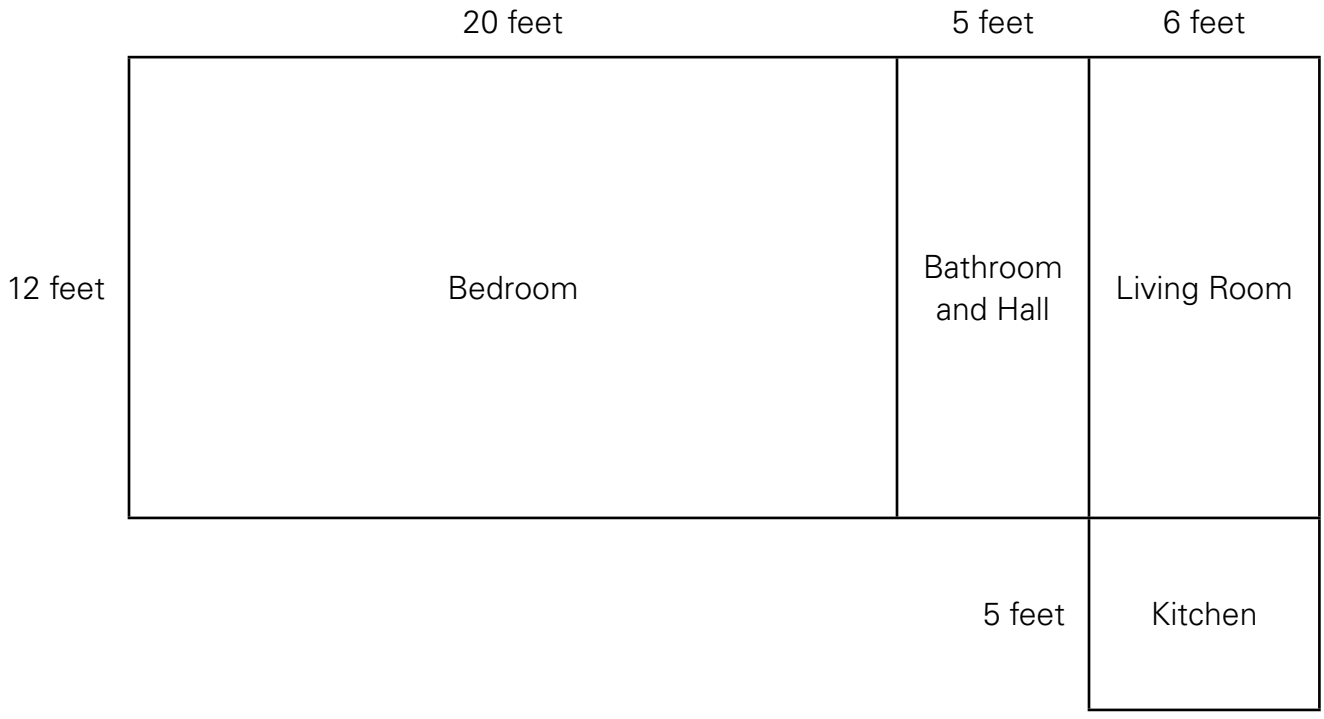
9. $2x(3x - y) - xy - y(x + y)$ _____

Would You Rather: Pick a Suite

Hotel Suite 1:

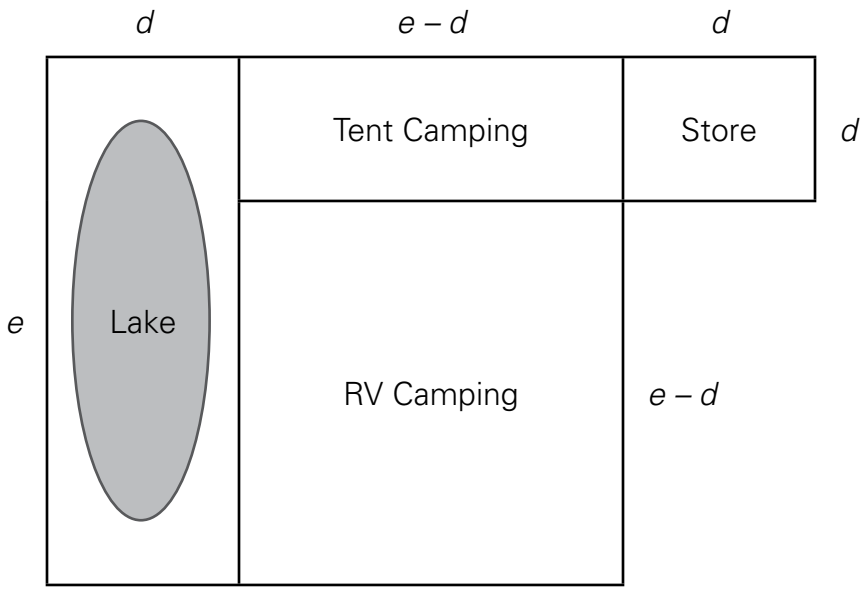


Hotel Suite 2:



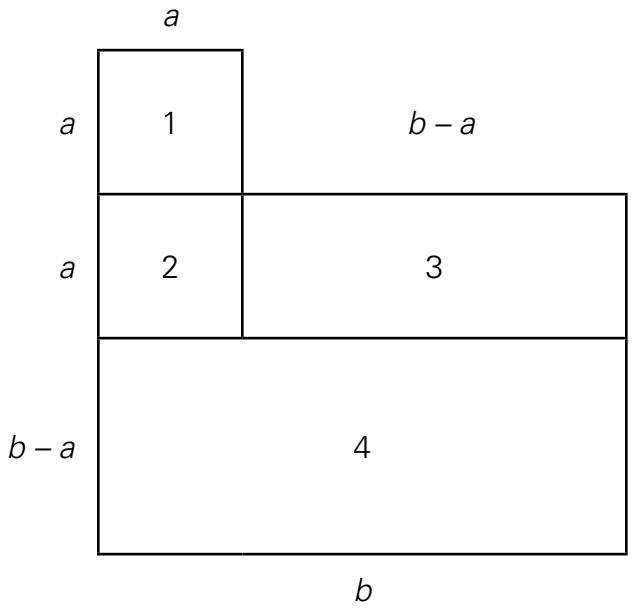
Multi-section Shape Diagrams

Campground Plan Diagram



Tyrell's mom's expression for total area: $ed + (e - d)(d) + d^2 + (e - d)(e - d)$

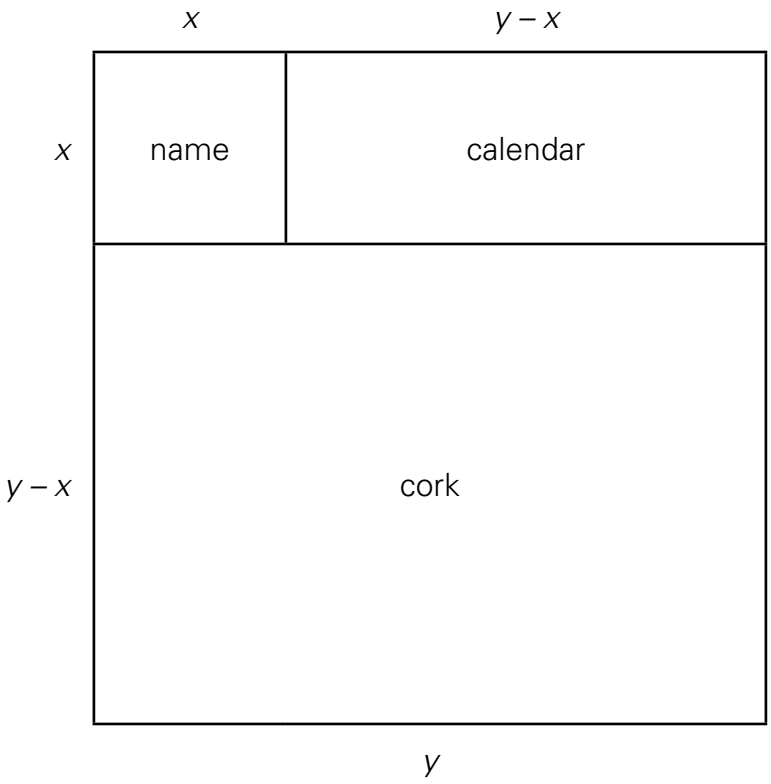
Josh's Camp Design



Given equation for total area: $a^2 + a^2 + a(b - a) + b(b - a)$

Bulletin Board Area

Bulletin Board Diagram



Bulletin Board Area: $x^2 + y(y - x) + x(y - x)$

Assessment

Unit 5 Assessment

1. $(13s + 82) + (15s + 138)$

a. Draw a tape diagram to model the problem.

b. Add. Show your work combining like terms and simplifying the expressions.

$(13s + 82) + (15s + 138) =$ _____

2. Subtract. Show your work.

$(16m + 38) - (9m + 2) - (47m + 15) =$ _____

3. Identify the greatest common factor and then use the distributive property to factor the expression.

GCF: _____

$42x + 35 =$ _____

4. Use the expression $12p + 16p$.

a. Draw a tape diagram for each term in the expression. Use the GCF to divide each tape into equal groups. Circle the groups.

b. Write a factored expression to represent this situation. _____

5. Use the expressions $4(6d + 13d + 2d)$ and $24d + 52d + 8d$.

a. Expand the first expression using the distributive property of multiplication.

$4(6d + 13d + 2d) =$ _____

b. Is $4(6d + 13d + 2d)$ equivalent to $24d + 52d + 8d$? _____

6. Use the distributive property of multiplication to expand the expression. Show your work.

$6(4n + 15 + 9w) =$ _____

7. Farad has been given the expressions $2(5a + 7)$ and $3(3a + 4) + (a + 2)$.

a. Draw a tape diagram for each expression and simplify.

$2(5a + 7)$

$3(3a + 4) + (a + 2)$

b. Are Farad's expressions equivalent? Explain. _____

_____

_____

8. Are the expressions $6(4c + 3)$ and $8(3c + 2)$ equivalent? _____

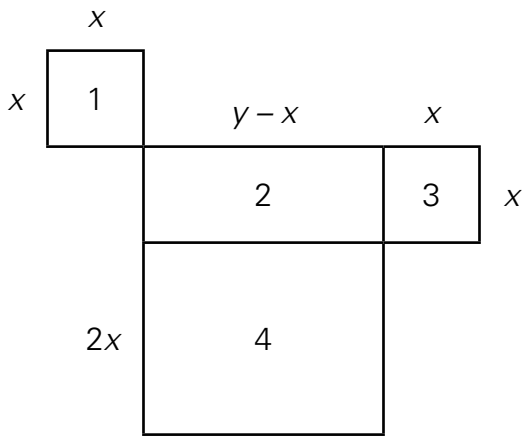
Show your work.

9. Use the grouping of like terms and properties of operations to find an equivalent expression. Show your work.

$5d(d - e) + d^2 + 2(de) =$ _____

10. Kirk is designing a multi-section picture frame to hold different photos. His design is shown. He was given the following expression:

$x^2 + x(y - x) + x^2 + 2x(y - x)$



a. Draw a tape diagram and use properties of operations to find an equivalent expression for $x^2 + x(y - x) + x^2 + 2x(y - x)$.

The equivalent expression is _____.

b. Group like terms and use the properties of operations alone (without a tape diagram) to find the equivalent expression. Show your work.

Unit 5 Cumulative Review

1. Gary has started a new exercise program. For every 3 miles he runs, he needs to do 25 pushups.

a. Write the ratio of miles run to pushups three different ways.

_____ _____ _____

b. How many pushups will Gary need to do if he runs 12 miles? _____

2. Multiply.

$-2\frac{5}{8} \times \frac{1}{2} =$ _____

3. Reyna is mixing paint for her art project. She is combining blue paint and green paint to make a shade of aqua. The aqua recipe includes 5 teaspoons of blue paint and 9 teaspoons of green paint.

a. Write the ratio for the blue to green paint in teaspoons. _____

b. Find the constant of proportionality of teaspoons of blue paint per teaspoons of green paint.

The constant of proportionality is _____.

c. How many teaspoons of blue paint will Reyna need for 36 teaspoons of green paint?

_____

4. For the inequality $d < 12$, circle the values that could be possible solutions and cross out the values that are impossible solutions.

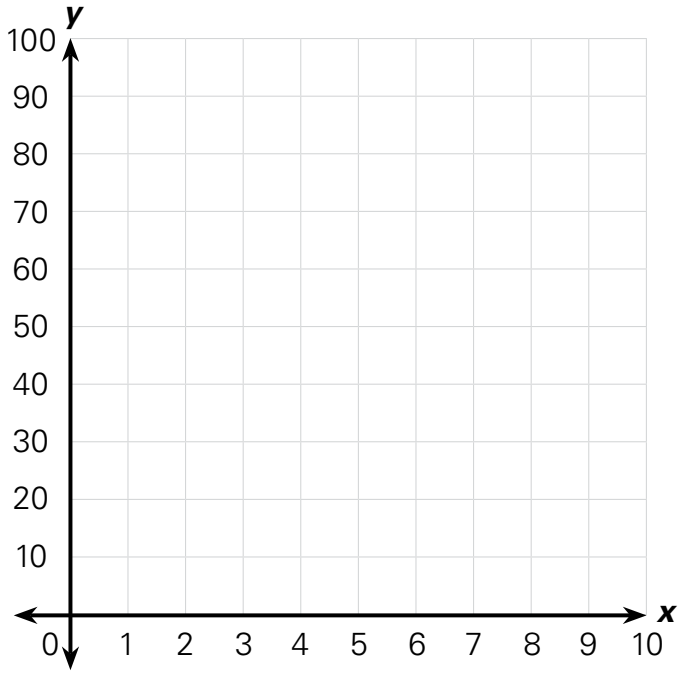
0 12 –14 6 –5 3 18 20

5. Marcus read 54 pages in 9 days.

a. What equation represents pages read per day? Let p = pages and t = time in days.

_____

b. On the coordinate plane, create a graph of Marcus' page reading. Remember to label the axes.



c. On your graph, what does the point (4, 24) represent? _____

6. In her spare time, Alicia builds bird feeders. She can build 8 bird feeders in 4 hours.

a. What is Alicia's unit rate in bird feeders per hour? _____

b. Draw a tape diagram to represent the unit rate.

c. How many bird feeders can Alicia make in 20 hours? Let b = bird feeders. _____

7. What is the unit rate for $\frac{2/5}{7/25}$? Show your work.

The unit rate is _____.

8. The Snack Shack is running a special this month. The entire month, each milkshake is only $2.

a. What is the unit rate for milkshakes this month in dollars per milkshake? _____

b. Write an equation in the form $y = mx$ to show how much it will cost for any number of milkshakes. _____

9. A fish is 32 feet below the surface of the water. It swims down 76 more feet. What is its depth now? _____

10. Complete the ratio table.

Butter (tablespoons)	Sugar (teaspoons)
0	0
	12
3	18
4	
	30
6	
7	
	48

The unit rate for this ratio table is _____.

11. Subtract.

7 – 18 = _____

12. Use the distributive property of multiplication to expand the expression $12(5q + 10q + 17q)$.

_____

13. What is 65% of 800? Show your work.

65% of 800 is _____.

14. Solve for *m*. Show your work.

$m + 45 = 18$

$m =$ _____

15. Divide.

$-64 \div 4 =$ _____